20. Exports and Imports from Production and Expenditure Approaches and Associated Price Indices Using an Artificial Data Set

A. Introduction

20.1 Chapters 16 to 18 outlined alternative price index number formulas, the factors that determine the nature and extent of differences between their results, and the criteria for choosing among them. The criteria for choosing among the formulas included the fixed basket, axiomatic, stochastic, and economic theoretic approaches. The first purpose of this chapter is to give the reader some idea of how much the major indices defined in the previous chapters differ using an artificial data set consisting of prices and quantities for 6 commodities over 5 periods. The period can be thought of as somewhere between a year and 5 years. The trends in the data are generally more pronounced than one would see in the course of a year. The 6 commodities can be thought of as the deliveries to the domestic final demand sector of all industries in the economy.

20.2 Chapter 15 showed how the nominal values of, and thus price indices for, exports and imports fitted into the *1993 System of National Accounts*. Particular emphasis was given to the role of price indices as deflators for estimating volume changes in gross domestic product (GDP) by the expenditure approach. The second purpose of this chapter is outline how price indices for exports and imports can be defined and reconciled from the expenditure and production approaches to estimating GDP. Indeed the illustrative data used to outline and demonstrate differences in the results from different index number formulas will be applied not only to export and import price indices, but also to price indices for the constituent aggregates of GDP from both the expenditure and production approaches.

20.3 There is a clear relationship in the *System* between GDP estimates from these two approaches that derives from the well-known identity between the sources and uses of goods and services as depicted in the *System'*s goods and services account. On the left-hand-side of the account the total amount of *resources* available to the domestic economy consist of the sum of outputs and imports and this is equal, on the right hand side, to the total amount *used* for consumption, investment, and exports, i.e.:

(20.1) $O + M + (t\text{-}s) = IC + C + I + G + X$

where O is the value of output of goods and services, M is the value of imports of goods and services, IC the value of goods and services used in the production process (intermediate consumption), C is final consumption expenditure of households and NPISH[1], I is gross capital formation, G is final consumption expenditure of government , and X is the value of exports of

[1] Non-profit institutions serving households—legal entities principally engaged in the production of non-market services for households whose main resources are voluntary contributions by households, such as charities and trade unions.

goods and services. Goods and services emanate from their original producers, either resident producers or producers abroad, for use by either resident users or users abroad.

20.4 Moving intermediate consumption from the right-hand-side of the account to the left, as a negative resource, while moving imports from the left to the right as a negative use, results in both sides now summing to GDP. The left hand side presents the production approach and the right hand side presents the expenditure approach.

(20.2) O - IC+ (t-s) = C + I + G + (X – M)

20.5 While exports and imports are explicitly identified in the expenditure approach this is not the case in the production approach. The production account in the *System* does not break O and IC down into output to the domestic market, Od, and rest of the world, Orow, and similarly for intermediate consumption, ICd and ICrow, however there remains an obvious decomposition of:

(20.3) (Od+Orow) – (ICd+ICrow) + (t-s) = C + I + G + (X – M)

However, it should be noted that while some of the value of exports and imports will be undertaken directly by households, NPISH, and government, for example tourist/cross border shopping, a large proportion of X and M can be represented as Orow and ICrow respectively on the production side.

20.6 GDP estimated from the *production approach* is based on the *value added* to the value of goods and services used in the production process (intermediate consumption), IC, to generate the value of output, O. GDP can be thought of as being equal to the sum of the value added produced by all institutional units resident in the domestic economy. The output is valued at basic prices to exclude taxes and subsidies, t and s, on products, while intermediate consumption and all other aggregates in the above equations are valued at purchasers' prices to include them. Taxes less subsidies on products need to be added back to value added to ensure that the values of what are supplied and used are equal. GDP is defined from the production approach on the right hand side of (20.3), therefore, as the sum of value added by resident producers plus the value of taxes less subsidies on products.

20.7 The *expenditure approach* involves summing the values of final consumption, gross capital formation (i.e., gross fixed capital formation, changes in inventories, and net acquisition of valuables[2]). These final expenditures do not properly represent all domestic economic activity since they exclude that directed to non-residents, i.e. exports, and include that arising from of non-residents, i.e. imports: exports and imports are respectively added to

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[2] This third category of capital formation, net acquisition (i.e., acquisitions less disposals) of valuables was introduced in the *1993 SNA*. Valuables (such as precious stones and metals, and paintings) are used as "stores of value" and not for consumption or production.

and subtracted from final consumption expenditure and capital formation on the right hand side of (20.2) and (20.3) to estimate GDP.

20.8 It is apparent from (20.1) that transactions in the *System* are treated as either a use or resource. It is also the case that both ends of a transaction are included in the *System*, that is the accounts include a transaction as outgoing "use" from one part of the *System* and an incoming "resource" to another. Transactions included in the *System* must capture, and close all the flows, not just those between two resident units. Thus for exports to be a "use" on the right hand side of (20.1) it must be used by non-residents and for imports to a "supply" on the left hand side of (20.1) it must be a supply by non-residents. It follows that the conceptualization of (X-M) on the right hand side of (20.2), which contributes to the expenditure estimate of GDP, requires X and M to be treated conceptually from the non-resident's perspective while the remaining transactions on output, intermediate consumption, final consumption, and investment are considered from the resident's perspective. Chapter 15 provide more detail on this distinction within the framework of *1993 System of National Accounts*, Chapter 18 considered the implications for the economic theory of index numbers and Dridi and Zieschang (2004) provide yet further detail on the non-resident's perspective to the treatment of exports and imports.

20.9 It is stressed that the perspective taken for nominal values of GDP carries over to volume estimates and thus the export and import price index numbers used to derive the volume estimates. There is thus a need to consider the representation of GDP estimates in equation (20.3) with attention given to the treatment of exports and imports from both the production and expenditure approaches. As noted above, the treatment of exports and imports from the expenditure approach is according to a non-resident's perspective as outlined in Dridi and Zieschang (2004) while the treatment of exports and imports from the production approach is according to a resident's perspective. The expenditure approach from a non-resident's perspective would be appropriate for import and export price index numbers for deflation of their nominal counterparts on the right hand side of (20.2) in the *System*. The production approach from a resident's perspective would be appropriate for import and export price and volume series used for the analysis of (the resident country's) productivity change, changes in the terms of trade, and transmission of inflation.

20.10 It was noted above that a second purpose of this chapter is outline how price indices for exports and imports can be defined and reconciled from the expenditure and production approaches to estimating GDP. Further, that the illustrative data used in this Chapter to outline and demonstrate differences in the results from different formulas were to be applied not only to export and import price indices, but also to the constituent aggregates of GDP from both expenditure and production approaches, as indicated in equations (20.2) and (20.3). The representation of the two approaches in equation (20.3) is simplistic as the aggregates are not broken down by commodity detail. Table 15.1 in the main production accounts of the *SNA 1993* does not elaborate on which industries are actually using the imports or on which industries are actually doing the exporting by commodity.[3] Hence, the main additions to the *SNA 1993* Chapter 15 for XMPI Manual purposes are to add tables to the main production accounts that provide industry by commodity detail on exports and imports. With these additional tables on the industry by commodity allocation of exports and imports, the resident's approach to collecting export and import price indexes can be embedded in the SNA framework.

20.11 The focus of Section B is to outline an expanded production account that includes its constituent commodity detail, B.1, with the effects of taxes and subsidies included, B.2, and, in B.3, expanded input output tables and a reconciliation of the expanded production and expenditure GDP estimates. The account given is of a simplified economy and Chapter 15 sets out the framework more formally.

20.12 Section C provides illustrative data for the framework in Section B. The data will be used to not only illustrate how export and import price indices differ according to the index number formula used, but to embed the price index numbers for exports and imports into an illustrative framework for the deflation of the constituent aggregates of GDP and GDP as a whole from both the expenditure and production frameworks.

20.13 Illustrative The data on domestic final demand deliveries are provided for a model of production. There are three industries in the economy and in principle, each industry could produce and use combinations of the 6 final demand commodities plus an additional imported "pure" intermediate input that is not delivered to the domestic final demand sector. In section C below, the basic industry data are listed in the input output framework that was explained in Section B; i.e., there are separate Supply and Use matrices for domestically produced and used commodities and for internationally traded commodities.

20.14 To summarize: price and quantity data for three industrial sectors of the economy are presented in section C. This industrial data set is consistent with the domestic final demand data set outlined in section D. A wide variety of indices are computed in section D using this final demand data set.

20.15 Section E constructs domestic gross output, export, domestic intermediate input and import price indices for the aggregate production sector. Only the Laspeyres, Paasche, Fisher and Törnqvist fixed base and chained formulas are considered in section E and subsequent sections since these are the formulas that are likely to be used in practice. The data used in sections E, F and G are at producer prices; this means that basic prices are used for domestic outputs and exports and purchasers' prices are used for imports and domestic intermediate inputs.

20.16 In sections F.1-F.3, value added price deflators are constructed for each of the three industries. A national value added deflator is constructed in section F.4.

[3] It should be noted that *SNA 1993* does have a recommended optional Table 15.5 which is exactly suited to our present needs; i.e., this table provides the detail for imports by commodity and by industry. However, *SNA 1993* does not provide a recommendation for a corresponding commodity by industry table for exports.

20.17 Section G compares alternative two stage methods for constructing the national value added deflator. This deflator can be constructed in a single stage by aggregating the detailed industry data (and this will be done in section F.4) or it can be constructed in two stages by either aggregating up the three industry value added deflators (see section F.1) or by aggregating up the gross output, export, intermediate input and import price indices that were constructed in section F (see section G.2). These two stage national value added deflators are compared with each other and their single stage counterpart.

20.18 Finally, in section H, final demand purchasers' prices are used in order to construct domestic final demand price indices (section H.1), export price indices (section H.2) and import price indices (section H.3). In section H.4, national GDP price deflators are constructed using final demand prices. Finally, in section H.5, the national value added deflator, which is constructed using producer prices, is compared to the national GDP deflator, which is constructed using final demand prices. This section also shows how these two national deflators can be reconciled with each other, provided that detailed industry by commodity data on commodity taxes and subsidies are available.

B. Expanded Production Accounts for the Treatment of International Trade Flows

B.1 Introduction

20.19 In order to set the stage for the economic approaches to the import and export price indexes from the resident's perspective, it is necessary to provide a set of satellite accounts for the production accounts in the *System of National Accounts 1993*. It turns out that the *SNA* treatment of the production accounts[4] is not able to provide an adequate framework for introducing a producer based economic theory of the export and import price indexes that would be analogous to the economic PPI indexes that were introduced in the Producer Price Index Manual.[5]

20.20 There is an extensive national income accounting literature on how to measure the effects of changes in the terms of trade (the export price index divided by the import price index) on national welfare.[6] However, Kohli (1978) (1991)[7] observed that most international trade flows through the production sector of the economy and hence a natural starting point for developing import and export price indexes is to imbed exports and imports in the production accounts of an economy.

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[4] See Chapter XV of Eurostat, IMF, OECD, UN and the World Bank, (1993).

[5] See ILO *et al.* (2004).

[6] See Diewert and Morrison (1986) for references to this early literature.

[7] This production theory approach to modeling trade flows was also used by Diewert (1974a; 142-146), Woodland (1982), Diewert and Morrison (1986), Alterman, Diewert and Feenstra (1999) and Feenstra (2004; 64-98).

20.21 There are *two main differences* between the production accounts that will be introduced in this Chapter and the Production Accounts that are described in the *SNA 1993*:

- The commodity classification is expanded to distinguish between domestically used and produced goods and services and internationally traded goods and services that flow through the production sector.
- The single supply of products and single use of products matrices (the Supply and Use matrices) that appear in the *SNA 1993*[8] are in principle replaced by a series of Supply and Use matrices so that the bilateral transactions of each industry with each one of the remaining industries can be distinguished.[9]

There will also be some discussion of the role of transport in the Input Output tables since imports and exports of goods necessarily involve some use of transportation services.

B.2 Expanded Input Output Accounts with No Commodity Taxation

20.22 In this section, a set of production accounts is developed for the production sector of an economy that engages in international trade. In order to simplify the notation, there are only three industries and three commodities in the commodity classification. Industry G (the goods producing industry) produces a composite good (commodity G), Industry S produces a composite service that excludes transportation services (commodity S) and Industry T provides transportation services (commodity T). In addition to trading goods and services between themselves, the three industries also engage in transactions with two final demand sectors:

- Sector F, the domestic final demand sector and
- Sector R, the Rest of the World sector.

20.23 The three industries deliver goods and services to the domestic final demand sector F.[10] They also deliver goods and services to the Rest of the World sector R[11] and they utilize deliveries from the Rest of the World sector as inputs into their production processes.[12]

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[8] See Table 15.1 in *SNA 1993*.

[9] This is what was done in Chapter 18 of the *Producer Price Index Manual*; see ILO *et al*. (2004; 463-507)

[10] These deliveries correspond to the familiar C + I + G final demand sectors.

[11] These deliveries correspond to exports, X.

[12] These deliveries correspond to imports, M.

20.24 The structure of the flows of goods and services between the 3 production sectors and the two final demand sectors will be shown by 4 value flow matrices, Tables 20.1-20.4 below.

20.25 Table 20.1 below shows the value of the *gross output deliveries* to the domestic final demand sector F as well as the deliveries of each industry to the remaining two industries: it is the *domestic supply matrix* or *domestic gross output by industry and commodity matrix* for a particular period of time. The Industry G, S and T columns list the sales of goods and services to all domestic demanders for each of the three commodities.

Table 20.1: Domestic Supply Matrix in Current Period Values

	Industry G	**Industry S**	**Industry T**
G	$p_G^{GS}y_G^{GS}+p_G^{GT}y_G^{GT}+p_G^{GF}y_G^{GF}$	0	0
S	0	$p_S^{SG}y_S^{SG}+p_S^{ST}y_S^{ST}+p_S^{SF}y_S^{SF}$	0
T	0	0	$p_T^{TG}y_T^{TG}+p_T^{TS}y_T^{TS}+p_T^{TF}y_T^{TF}$

20.26 The value sum in row and column G, $p_G^{GS}y_G^{GS} + p_G^{GT}y_G^{GT} + p_G^{GF}y_G^{GF}$, corresponds to the revenues received by the goods producing sector from its sales of good G to the service sector, $p_G^{GS}y_G^{GS}$ where p_G^{GS} is the price of sales of good G to sector S and y_G^{GS} is the corresponding quantity sold[13], plus the revenues received by the goods producing sector from its sales of good G to the transportation sector, $p_G^{GT}y_G^{GT}$ where p_G^{GT} is the price of sales of good G to sector T and y_G^{GT} is the corresponding quantity sold, plus the revenues received by the goods producing sector from its sales of good G to the domestic final demand sector, $p_G^{GF}y_G^{Gf}$ where p_G^{GF} is the price of sales of good G to sector F and y_G^{GF} is the corresponding quantity sold. Similarly, the value sum in row and column S, $p_S^{SG}y_S^{SG} + p_S^{ST}y_S^{ST} + p_S^{SF}y_S^{SF}$, corresponds to the revenues received by the service sector from its sales of service S to the goods producing sector, the transportation sector and the domestic final demand sector. Finally, the value sum in row and column T, $p_T^{TG}y_T^{TG} + p_T^{TS}y_T^{TS} + p_T^{TF}y_T^{TF}$, corresponds to the revenues received by the transportation sector from its sales of transportation services T to the goods producing sector, the general services sector and the domestic final demand sector. It should be mentioned that these transportation prices are margin type prices; i.e., they are the prices for delivering goods from one point to another.[14] Note also that p_G^{GS} will usually not equal p_G^{GT} or p_G^{GF} i.e., for a variety of reasons, the average selling price of the domestic good to the three sectors that demand the good will usually be different and a similar comment applies to the other commodity prices.[15] Unfortunately, this means that we

[13] Under the assumption that there are no quality differences between units of G, the appropriate price will be a unit value and the corresponding quantity will be the total quantity of G purchased by sector S during the period.

[14] For a more detailed analysis of how transport quantities are tied to shipments of goods from one sector to another, see Diewert (2006).

[15] Even if there is no price discrimination on the part of Industry G at any point in time, the price of good G will usually vary over the reference period and hence if the proportion of daily sales varies between the three

(continued)

cannot use a common price for a commodity across sectors to deflate the value flows in Tables A1 and A2 into volume quantity flows by commodity; i.e., basic prices that are constant across sectors will usually not exist. This is another reason why it is useful to extend the SNA 1993 production accounts.

20.27 Table 20.2 below shows the value of the purchases of *intermediate inputs* for each industry from domestic suppliers; it is the *domestic use matrix* or *domestic intermediate input by industry and commodity matrix*.

Table 20.2: Domestic Use Matrix in Current Period Values

	Industry G	**Industry S**	**Industry T**
G	0	$p_G^{GS}y_G^{GS}$	$p_G^{GT}y_G^{GT}$
S	$p_S^{SG}y_S^{SG}$	0	$p_S^{ST}y_S^{ST}$
T	$p_T^{TG}y_T^{TG}$	$p_T^{TS}y_T^{TS}$	0

Note that the value of purchases of goods from industry G by industry S, $p_G^{GS}y_G^{GS}$, is exactly equal to the value of sales of goods by industry G to industry S and this value appeared in the value of sales of goods G by industry G in Table 20.1. In fact, all of the domestic purchases of intermediate inputs listed in Table 20.2 have their domestic sales counterpart entries in Table 20.1.

20.28 The next table shows the value of the *gross output deliveries* to the Rest of the World final demand sector R; it is the *ROW supply matrix* or more simply, the *export by industry and commodity matrix*.

Table 20.3: Export or ROW Supply Matrix in Current Period Values

	Industry G	**Industry S**	**Industry T**
G	$p_{Gx}^{GR}x_G^{GR}$	0	0
S	0	$p_{Sx}^{SR}x_S^{SR}$	0
T	0	0	$p_{Tx}^{TR}x_T^{TR}$

20.29 The value sum in row and column G, $p_{Gx}^{GR}x_G^{GR}$, corresponds to the revenues received by the goods producing sector from its sales of good G to the Rest of the World sector, where p_{Gx}^{GR} is the price of sales of good G to sector R and x_G^{GR} is the corresponding quantity sold, or more simply, it is the value of exports by the goods producing sector to the

sectors, the corresponding period average prices for the three sectors will be different. The notation used here is unfortunately much more complicated than the notation that is typically used in explaining input output tables because it is not assumed that each commodity trades across demanders and suppliers at the same price. Thus the above notation distinguishes 3 superscripts or subscripts instead of the usual 2: 2 superscripts are required to distinguish the selling and purchasing sectors and one additional subscript is required to distinguish the commodity involved in each transaction. This type of setup was used in chapter 19 of ILO *et al.* (2004).

Rest of the World.[16] Similarly, $p_{Sx}{}^{SR}x_S{}^{SR}$ is the value of exports of services produced by the services sector and $p_{Tx}{}^{TR}x_T{}^{TR}$ is the value of exports of transportation services produced by the transportation sector. Note that not all of these transportation sector export revenues need be associated with the importation of goods into the domestic economy: some portion of these revenues may be due to the shipment of goods between two or more foreign countries.

20.30 Table 20.4 below shows the value of the purchases of *intermediate inputs* or *imports* from the Rest of the World for each industry by commodity; it is the *Import* or *ROW use matrix* or *ROW intermediate input by industry and commodity matrix*.

Table 20.4: Import or ROW Use Matrix in Current Period Values

	Industry G	**Industry S**	**Industry T**
G	$p_{Gm}{}^{GR}m_G{}^{GR}$	$p_{Gm}{}^{SR}m_G{}^{SR}$	$p_{Gm}{}^{TR}m_G{}^{TR}$
S	$p_{Sm}{}^{GR}m_S{}^{GR}$	$p_{Sm}{}^{SR}m_S{}^{SR}$	$p_{Sm}{}^{TR}m_S{}^{TR}$
T	$p_{Tm}{}^{GR}m_T{}^{GR}$	$p_{Tm}{}^{SR}m_T{}^{SR}$	$p_{Tm}{}^{TR}m_T{}^{TR}$

20.31 The value of imports in row and column G, $p_{Gm}{}^{GR}m_G{}^{GR}$, corresponds to the payments to the Rest of the World by the goods producing sector for its imports of goods, where $p_{Gm}{}^{GR}$ is the price of imports of good G to industry G and $m_G{}^{GR}$ is the corresponding quantity purchased, or more simply, it is the cost of imports of goods to the goods producing sector. Similarly, $p_{Sm}{}^{GR}m_S{}^{GR}$ is the value of imported services that are used in the goods producing sector and $p_{Tm}{}^{GR}m_T{}^{GR}$ is the value of imported transportation services that are used in the goods producing sector. Note that industry G may purchase transportation services from domestic or foreign suppliers and a similar comment applies to the purchases of transportation services by industries S and T. The imported value flows for industries S and T are similar to the corresponding import values for the goods producing industry.

20.32 The above four matrices are in terms of current period values. The corresponding *constant* period values or volume *matrices* can readily be derived from the matrices listed in Tables 20.1-20.4: simply drop all of the prices from the above matrices and the resulting matrices, which will have only quantities as entries in each cell, will be the corresponding constant dollar input output matrices. However, note that unless all prices are identical for each entry in each cell of a row, the correct volume entries will *not* be obtained in general by deflating each row of each matrix by a common price deflator. This observation means that statistical agencies who use the common deflator method to obtain volume input output tables from corresponding nominal input output tables may be introducing substantial errors into their estimates of volume value added by sector. In principle, each cell in a nominal use or make matrix will require a separate deflator in order to recover the corresponding correct volume entry.

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[16] We make the general convention that the last *non transportation* domestic establishment that handles an exported good is regarded as the sector which exports the good. If we did not make this convention, virtually all exported goods would be credited to the transportation sector. This convention is consistent with our treatment of transportation services as a margin industry.

20.33 The nominal value flow matrices defined by Tables 20.1-20.4 and their volume counterparts can be used to derive the traditional Supply and Use matrices that appear in Table 15.1 of the *System* of National Accounts 1993: the conventional Supply matrix is the sum of the matrices in 20.1 and 20.3 (the Domestic and ROW Supply matrices) while the conventional Use matrix is the sum of the matrices in 20.2 and 20.4 (the Domestic and ROW Use matrices). Finally, the matrix that is needed for export and import price indexes can be obtained by adding entries in Tables 20.1 and 20.3 and then subtracting the corresponding entries in Tables 20.2 and 20.4 in order to obtain a *net supply matrix* that gives the value of net commodity supply by commodity and by industry of origin.

20.34 The net supply matrix can be aggregated in two ways:

- By summing over columns along each row; the resulting value aggregates are *net supplies by commodity*, which are equal to domestic final demands plus exports less imports (*net final demands by commodity*), or
- By summing over rows down each column; the resulting value aggregates are equal to *value added by industry*.

20.35 It will be useful to list the aggregates that result by implementing the above two methods of aggregation using the entries in Tables 20.1-20.4. The 3 commodity final demand aggregates turn out to be the following value aggregates:[17]

$$(20.4)\ vf_G \equiv p_G^{GF}y_G^{GF} + p_{Gx}^{GR}x_G^{GR} - p_{Gm}^{GR}m_G^{GR} - p_{Gm}^{SR}m_G^{SR} - p_{Gm}^{TR}m_G^{TR};$$

$$(20.5)\ vf_S = p_S^{SF}y_S^{SF} + p_{Sx}^{SR}x_S^{SR} - p_{Sm}^{GR}m_S^{GR} - p_{Sm}^{SR}m_S^{SR} - p_{Sm}^{TR}m_S^{TR};$$

$$(20.6)\ vf_T = p_T^{TF}y_S^{SF} + p_{Tx}^{TR}x_S^{TR} - p_{Tm}^{GR}m_T^{GR} - p_{Tm}^{SR}m_T^{SR} - p_{Tm}^{TR}m_T^{TR}.$$

20.36 The three industry value added aggregates are defined as follows:

$$(20.7)\ va^G \equiv p_G^{GS}y_G^{GS} + p_G^{GT}y_G^{GT} + p_G^{GF}y_G^{GF} - p_S^{SG}y_S^{SG} - p_T^{TG}y_T^{TG} + p_{Gx}^{GR}x_G^{GR} - p_{Gm}^{GR}m_G^{GR} - p_{Sm}^{GR}m_S^{GR} - p_{Tm}^{GR}m_T^{GR};$$

$$(20.8)\ va^S \equiv p_S^{SG}y_S^{SG} + p_S^{ST}y_S^{ST} + p_S^{SF}y_S^{SF} - p_G^{GS}y_G^{GS} - p_T^{TS}y_T^{TS} + p_{Sx}^{SR}x_S^{SR} - p_{Gm}^{SR}m_G^{SR} - p_{Sm}^{SR}m_S^{SR} - p_{Tm}^{SR}m_T^{SR};$$

$$(20.9)\ va^T \equiv p_T^{TG}y_T^{TG} + p_T^{TS}y_T^{TS} + p_T^{TF}y_T^{TF} - p_G^{GT}y_G^{GT} - p_S^{ST}y_S^{ST} + p_{Tx}^{TR}x_T^{TR} - p_{Gm}^{TR}m_G^{TR} - p_{Sm}^{TR}m_S^{TR} - p_{Tm}^{TR}m_T^{TR}.$$

20.37 Note that each commodity final demand value aggregate, vf_G, vf_S and vf_T, is equal to the value of industry deliveries of each of the three commodities plus export deliveries less imports of the commodity to each of the three industrial sectors. Note also that it is not in general appropriate to set the price of say vf_G equal to the value of vf_G divided by the

[17] Each of the net supply aggregates defined by (20.4)-(20.6) does not have to be positive; for example, consider the case of an imported intermediate good that is not produced domestically. However the sum of the net supply aggregates will be substantially positive.

corresponding net deliveries of commodity G to final demand, $y_G^{GS}+y_G^{GT}+{}^{F}y_G^{GF} - y_S^{SG} - y_T^{TG} + x_G^{GR} - m_G^{GR} - m_S^{GR} - m_T^{GR}$, because differences in the prices that are attached to these quantities imply that there are implicit quality differences between these quantities. Thus index number theory should be used to aggregate the value flows on the right hand sides of (20.7)-(20.9). It is clear that index number theory must be used to construct a price and quantity for each of the value added aggregates, va^G, va^S and va^T, since by inspecting (20.7)-(20.9), it can be seen that each value added aggregate is a sum over heterogeneous commodities, some with positive signs associated with their quantities (these are the gross outputs produced by the industry) and some with negative signs (these are the foreign sourced and domestic intermediate inputs used by the industry).

20.38 1 The three final demand value aggregates defined by (20.7)-(20.9) can be summed and the resulting value aggregate is the GDP generated by the economy's production sector. Alternatively, the value added aggregates defined by (20.7)-(20.9) can also be summed and this sum will also equal GDP since these two methods of aggregation are simply alternative methods for summing over the elements of the net supply matrix. Thus the following equation must hold:

$$(20.10)\ GDP \equiv vf_G + vf_S + vf_T = va^G + va^S + va^T.$$

20.39 It is useful to use (20.10) which defines GDP as the sum of the value of final demands and substitute (20.7)-(20.9) into this definition in order to obtain the following expression for GDP after some rearrangement of terms:

$$\begin{aligned}(20.11)\ GDP &= vf_G + vf_S + vf_T \\ &= [p_G^{GF}y_G^{GF} + p_S^{SF}y_S^{SF} + p_T^{TF}y_S^{SF}] + [p_{Gx}^{GR}x_G^{GR} + p_{Sx}^{SR}x_S^{SR} + p_{Tx}^{TR}x_S^{TR}] \\ &\quad - [p_{Gm}^{GR}m_G^{GR} + p_{Gm}^{SR}m_G^{SR} + p_{Gm}^{TR}m_G^{TR} + p_{Sm}^{GR}m_S^{GR} + p_{Sm}^{SR}m_S^{SR} \\ &\quad + p_{Sm}^{TR}m_S^{TR} + p_{Tm}^{GR}m_T^{GR} + p_{Tm}^{SR}m_T^{SR} + p_{Tm}^{TR}m_T^{TR}] \\ &= [C+I+G] + [X] - [M].\end{aligned}$$

20.40 Note that the value aggregate, $p_G^{GF}y_G^{GF} + p_S^{SF}y_S^{SF} + p_T^{TF}y_S^{SF}$, corresponds to the value of domestic final demand, the value aggregate $p_{Gx}^{GR}x_G^{GR} + p_{Sx}^{SR}x_S^{SR} + p_{Tx}^{TR}x_S^{TR}$ corresponds to the value of exports and the value aggregate $p_{Gm}^{GR}m_G^{GR} + p_{Gm}^{SR}m_G^{SR} + p_{Gm}^{TR}m_G^{TR} + p_{Sm}^{GR}m_S^{GR} + p_{Sm}^{SR}m_S^{SR} + p_{Sm}^{TR}m_S^{TR} + p_{Tm}^{GR}m_T^{GR} + p_{Tm}^{SR}m_T^{SR} + p_{Tm}^{TR}m_T^{TR}$ corresponds to the value of imports. Thus (20.11) corresponds to the traditional final demand definition of GDP.[18]

20.41 Equation (20.10) shows that there are two alternative ways that data on transactions between the domestic production sector and the Rest of the World could be captured:

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[18] There are some minor complications due to the fact that small amounts of imports and exports may not pass through the domestic production sector; i.e., some tourist expenditures made abroad would not be captured by transactions within the scope of the domestic production sector and a similar comment applies to government expenditures made abroad.

- In the *final demand method*, information on the price and quantity for each category of import (export) would be obtained from the foreign supplier (demander). This is the *nonresident point of view*.
- In the *value added method*, information on the price and quantity of each type of import used by each industry and the price and quantity of each type of export produced by each industry would be obtained from the domestic producer. This is the *resident point of view*.

20.41 It is apparent that the practical compilation of trade price indices can be facilitated by developing the existing Producer Price Index (PPI) methodology[19]: the PPI methodology can be adapted to the export-import price index case to expand the commodity classification in order to make the distinction between a domestically sourced intermediate input and a foreign import and make the distinction between an output that is delivered to a domestic demander versus an output that is delivered to a foreign demander, which is an export. Of course, in practice, it may be difficult to make these distinctions. But distinct advantages of building on existing PPI computer routines, data collection and verification methods exist though there will remain the need to extend the sample of establishments and commodities to be representative of buyers and sellers from/to domestic and foreign markets.

20.42 At this point, it is useful to consider alternative methods for constructing volume measures for GDP originating in the domestic production sector. Thus suppose that data on production sector transactions are available for periods 0 and 1 and that price and quantity information is available for these two periods so that the data in Tables 20.1-20.4 are available and hence net supply matrices for the production sector can be calculated for periods 0 and 1. It can be seen that there are *three ways* that a volume or quantity index of net outputs for the production sector of economy could be calculated:

- Change the signs of the nonzero entries in the Domestic Use matrix defined by Table 20.2 and change signs of the nonzero entries in the ROW Use matrix defined by Table 20.4. Look at the nonzero cells in these two 3 by 3 matrices as well as the cells in the Suppy matrices defined in Tables 20.1 and 20.3. Collecting up all of these nonzero transactions, it can be seen that there are 27 distinct price times quantity transactions. If there is a negative sign associated with any one of these terms, that negative sign is attached to the quantity. Now apply normal index number theory to these 27 price times quantity components of the aggregate.
- Sum up the value added aggregates defined by (20.7)-(20.9). The resulting value added aggregate will have 27 separate price times quantity components. If a value component has a negative sign associated with it, then attach the negative sign to the quantity (so that all prices will always be positive). Now apply normal price index number formulas theory to these 27 price times quantity components of the aggregate.

[19] See ILO *et al.* (2004). Alterman, Diewert and Feenstra (1999) also used the value added methodology in their exposition of the economic approach to the export and import price indexes.

- Sum up the final demand value aggregates defined by (20.4)-(20.6). The resulting value of final demand aggregate will have 15 separate price times quantity components. If a value component has a negative sign associated with it, then attach the negative sign to the quantity. Now apply normal index number theory to these 15 price times quantity components of the aggregate.

20.76 It is evident that the quantity index or the volume estimate for GDP will be the same using methods 1 and 2 listed above since the two methods generate exactly the same set of 27 separate price times quantity components in the value aggregate. However, it is not evident that volume estimates for GDP based on method 3 will coincide with those generated using methods 1 and 2 since there are 27 price times quantity components to be aggregated when we use methods 1 or 2 compared to only 15 components when we use method 3.

20.77 Denote the 27 dimensional p (price) and q (quantity) vectors that correspond to the first detailed cell and value added methods for aggregating over commodities listed above as p^{va} and q^{va} respectively and denote the 15 dimensional p and q vectors that correspond to the third aggregation method over final demand components as p^{fd} and q^{fd} respectively.[20] Add a superscript t to denote these vectors evaluated at the data pertaining to period t. Then using (20.10), the inner products of each of these period t price and quantity vectors are equal in the same period since they are each equal to period t nominal GDP:[21]

(20.12) $p^{vat}\cdot q^{vat} = p^{fdt}\cdot q^{fdt}$; $\quad t = 0,1.$

20.78 What is not immediately obvious is that the inner products of the two sets of price and quantity vectors are also equal if the price vectors are evaluated at the prices of one period and the corresponding quantity vectors are evaluated at the quantities of another period; i.e., for periods 0 and 1, the following equalities hold:[22]

(20.13) $p^{va1}\cdot q^{va0} = p^{fd1}\cdot q^{fd0}$;
(20.14) $p^{va0}\cdot q^{va1} = p^{fd0}\cdot q^{fd1}$.

20.79 Laspeyres and Paasche quantity indexes that compare the quantities of period 1 to those of period 0 can be defined as follows:

(20.15) $Q_L^{va}(p^{va0},p^{va1},q^{va0},q^{va1}) \equiv p^{va0}\cdot q^{va1}/p^{va0}\cdot q^{va0}$;

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[20] All prices are positive but if a quantity is an input, it is given a negative sign.

[21] Notation: $p\cdot q \equiv \sum_{n=1}^{N} p_n q_n$ where p and q are N dimensional vectors with components p_n and q_n respectively.

[22] The proof follows using the additivity of the inner products and the exact matching of a domestic intermediate input transaction to a corresponding domestic output transaction. Diewert (2006; 293-294) used this method of proof, drawing on prior discussions on these issues with Kim Zieschang. Moyer, Reinsdorf and Yuskavage (2006) derived similar results but under the assumption that commodity prices were constant across industries.

$$Q_L^{fd}(p^{fd0},p^{fd1},q^{fd0},q^{fd1}) \equiv p^{fd0}\cdot q^{fd1}/p^{fd0}\cdot q^{fd0} ;$$

$$(20.16)\quad Q_P^{va}(p^{va0},p^{va1},q^{va0},q^{va1}) \equiv p^{va1}\cdot q^{va1}/p^{va1}\cdot q^{va0} ;$$
$$Q_P^{fd}(p^{fd0},p^{fd1},q^{fd0},q^{fd1}) \equiv p^{fd1}\cdot q^{fd1}/p^{fd1}\cdot q^{fd0} .$$

20.80 Using (20.12), (20.14) and definitions (20.15), it can be seen that the two Laspeyres volume indexes are equal:

$$(20.17)\ Q_L^{va}(p^{va0},p^{va1},q^{va0},q^{va1}) = Q_L^{fd}(p^{fd0},p^{fd1},q^{fd0},q^{fd1}).$$

20.81 Using (20.12), (20.13) and definitions (20.17), it can be seen that the two Paasche volume indexes are equal:

$$(20.18)\ Q_P^{va}(p^{va0},p^{va1},q^{va0},q^{va1}) = Q_P^{fd}(p^{fd0},p^{fd1},q^{fd0},q^{fd1}).$$

20.82 Since a Fisher ideal quantity index is the square root of the product of a Laspeyres and Paasche quantity index, it can be seen that (20.17) and (20.18) imply that all three Fisher quantity indexes, constructed by aggregating over Input Output net supply table cells or by aggregating over industry value added components (which is equivalent to aggregating over net supply table cells) or by aggregating over final demand components, are equal; i.e., we have:

$$(20.19)\ Q_F^{va}(p^{va0},p^{va1},q^{va0},q^{va1}) = Q_F^{fd}(p^{fd0},p^{fd1},q^{fd0},q^{fd1}).$$

20.83 The equality between the two methods for constructing volume estimates that is reflected in equations (20.17)-(20.19) could provide a potentially useful check on a statistical agency's methods for constructing aggregate volume GDP measures.

20.84 The above results extend to more complex input output frameworks provided that all transactions between each pair of sectors in the model are accounted for in the model.

20.85 The equality (20.19) between the two methods for constructing an aggregate volume index for GDP using the Fisher quantity index as the index number formula can be extended to the case where the implicit Törnqvist quantity index is used as the index number formula. In this case, the value aggregates are deflated by the Törnqvist price index and by writing out the formulae, it is straightforward to show that $P_T^{va}(p^{va0},p^{va1},q^{va0},q^{va1})$, the Törnqvist price index using the 27 price times quantity components in the value added aggregate, is equal to $P_T^{fd}(p^{fd0},p^{fd1},q^{fd0},q^{fd1})$, the Törnqvist price index using the 15 price times quantity components in the final demand aggregate.[23]

20.86 It is well known that the Laspeyres and Paasche quantity indexes are consistent in aggregation. Thus if Laspeyres indexes of volume estimates of value added by industry are

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[23] This observation was made in the PPI Manual and was confirmed by numerical computations; see ILO *et al.* (2004; 505-506).

constructed in the first stage of aggregation and the resulting industry prices and quantities are used as inputs into a second stage of Laspeyres aggregation, then the resulting two stage Laspeyres quantity index is equal to the corresponding single stage index, $Q_L^{va}(p^{va0},p^{va1},q^{va0},q^{va1})$. Similarly, if Paasche volume indexes of value added by industry are constructed in the first stage of aggregation and the resulting industry prices and quantities are used as inputs into a second stage of Paasche aggregation, then the resulting two stage Paasche quantity index is equal to the corresponding single stage index, $Q_P^{va}(p^{va0},p^{va1},q^{va0},q^{va1})$.[24] Unfortunately, the corresponding result does not hold for the Fisher index. However, the two stage Fisher quantity index usually will be quite close to the corresponding single stage index, $Q_F^{va}(p^{va0},p^{va1},q^{va0},q^{va1})$.[25]

In the following section, commodity taxes are introduced into the Supply and Use matrices.

B.3 Input Output Accounts with Commodity Taxation and Subsidization

20.87 Consider again the production model that corresponds to Tables 20.1-20.4 in the previous section but it is now assumed that there is the possibility of a commodity tax (or subsidies) falling on the output of each industry and on the intermediate inputs used by each industry. Assume that the *producing industry* collects these commodity taxes and remits them to the appropriate level of government. These indirect commodity taxes will be introduced into each of the Tables listed in the previous section.

The counterpart to Table 20.1 is now Table 20.5.

Table 20.5: Domestic Supply Matrix in Current Period Values with Commodity Taxes

	Industry G	Industry S	Industry T
G	$(p_G^{GS}-t_G^{GS})y_G^{GS}$ $+(p_G^{GT}-t_G^{GT})y_G^{GT}$ $+(p_G^{GF}-t_G^{GF})y_G^{GF}$	0	0
S	0	$(p_S^{SG}-t_S^{SG})y_S^{SG}$ $+(p_S^{ST}-t_S^{ST})y_S^{ST}$ $+(p_S^{SF}-t_S^{SF})y_S^{SF}$	0
T	0	0	$(p_T^{TG}-t_T^{TG})y_T^{TG}$

[24] A word of warning is in order if two stage aggregation is used: *the value aggregates in the first stage of aggregation must be of the same sign.* If they are not of the same sign, index number theory will fail. Thus it is not recommended that a first stage aggregate equal to exports minus imports be constructed, since the value of net exports could be positive in period 0 and negative in period 1. A similar problem arises if it is attempted to construct an index of real inventory change since the sign of the value aggregate can change from period to period. Diewert (2005) provides some examples of index number failure in the inventory change context but his analysis is applicable more generally.

[25] See Diewert (1978; 889) and Hill (2006). However, using an artificial data set, in Chapter 19 below, it will be shown that the two stage Fisher value added index is not close to its single stage counterpart so some caution must be used in aggregating value added across industries in a two stage aggregation procedure.

$+(p_T{}^{TS}-t_T{}^{TS})y_T{}^{TS}$
$+(p_T{}^{TF}-t_T{}^{TF})y_T{}^{TF}$

20.88 The quantity of goods delivered to the service sector is $y_G{}^{GS}$ as before and the service sector pays industry G the price $p_G{}^{GS}$ for each unit of G that was delivered. However, Industry G must remit the per unit[26] commodity tax[27] $t_G{}^{GS}$ of the per unit revenue $p_G{}^{GS}$ to the Government sector and so Industry G receives only the revenue $p_G{}^{GS}-t_G{}^{GS}$ for each unit of good G sold to Industry S. The interpretation of the other prices and commodity taxes that occur in Table 20.5 is similar.

20.89 The Domestic Use matrix in current period values is still defined by the entries in Table 20.2. This matrix remains unchanged with the introduction of commodity taxes and subsidies. This is because the domestic taxes and subsidies are assumed to be on the output of the producer. Had they been paid by the domestic purchaser on intermediate consumption they would appear here as part of the purchase price.

20.90 The Rest of the World Supply matrix or export by industry and commodity matrix defined earlier by Table 20.3 is now replaced by Table 20.6.

Table 20.6: Export or ROW Supply Matrix in Current Period Values with Export Taxes

	Industry G	**Industry S**	**Industry T**
G	$(p_{Gx}{}^{GR}-t_{Gx}{}^{GR})x_G{}^{GR}$	0	0
S	0	$(p_{Sx}{}^{SR}-t_{Sx}{}^{SR})x_S{}^{SR}$	0
T	0	0	$(p_{Tx}{}^{TR}-t_{Tx}{}^{TR})x_T{}^{TR}$

20.91 To interpret the entries in Table 20.6, consider the entries for commodity G and industry G. Industry G still gets the revenue $p_{Gx}{}^{GR}x_G{}^{GR}$ for its deliveries of goods to foreign purchasers from these purchases but if the government sector imposes a specific export tax equal to $t_{Gx}{}^{GR}$ per unit of exports, then Industry G only gets to keep the amount $p_{Gx}{}^{GR}-t_{Gx}{}^{GR}$ per unit sale instead of the full final demander price $p_{Gx}{}^{GR}$. If however, $t_{Gx}{}^{GR}$ is negative, then the government is subsidizing the export of goods and hence the subsidized price that the producer faces, $p_{Gx}{}^{GR}-t_{Gx}{}^{GR}$, is actually higher than the final demander price $p_{Gx}{}^{GR}$. The interpretation of the industry S and commodity S and industry T and commodity T entries are similar.

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[26] Thus the commodity taxes are modeled as specific taxes rather than ad valorem taxes. This is not a restriction on the analysis since ad valorem taxes can be converted into equivalent specific taxes in each period.

[27] If the sales of commodity G are being subsidized by the Government sector, then the tax level per unit $t_G{}^{GS}$ will be negative instead of being positive. It is assumed that the after tax prices of the form $p_G{}^{GS}-t_G{}^{GS}$ are always positive. In a more detailed model, per unit commodity subsidies could be explicitly introduced instead of the present interpretation of $t_G{}^{GS}$ as a specific *net* (tax less subsidy) commodity tax.

20.92 The Rest of the World Use matrix or the import matrix by industry and commodity defined by Table 20.4 in the previous section is now replaced by Table 20.7.

Table 20.7: Import or ROW Use Matrix in Current Period Values with Import Taxes

	Industry G	**Industry S**	**Industry T**
G	$(p_{Gm}^{GR}+t_{Gm}^{GR})m_G^{GR}$	$(p_{Gm}^{SR}+t_{Gm}^{SR})m_G^{SR}$	$(p_{Gm}^{TR}+t_{Gm}^{TR})m_G^{TR}$
S	$(p_{Sm}^{GR}+t_{Sm}^{GR})m_S^{GR}$	$(p_{Sm}^{SR}+t_{Sm}^{SR})m_S^{SR}$	$(p_{Sm}^{TR}+t_{Sm}^{TR})m_S^{TR}$
T	$(p_{Tm}^{GR}+t_{Tm}^{GR})m_T^{GR}$	$(p_{Tm}^{SR}+t_{Tm}^{SR})m_T^{SR}$	$(p_{Tm}^{TR}+t_{Tm}^{TR})m_T^{TR}$

20.93 As in Table 20.4, Industry G imports m_G^{GR} units of goods from foreign suppliers and pays these foreign suppliers the amount $p_{Gm}^{GR}m_G^{GR}$. However, if t_{Gm}^{GR} is positive (the usual case), then the government imposes a specific set of tariffs and indirect taxes on each unit imported equal to t_{Gm}^{GR} and hence Industry G faces the higher price $p_{Gm}^{GR}+t_{Gm}^{GR}$ for each unit of good G that is imported.[28] The interpretation of the industry S and commodity S and industry T and commodity T entries are similar.

20.94 The *volume industry Supply and Use matrices* that correspond to the nominal Supply matrices defined by Tables 20.5 and 20.6 and nominal Use matrices defined by Tables 20.2 and 20.7 can be obtained from their nominal counterparts after deleting all of the price and tax terms. For completeness, these volume Supply and Use matrices are listed below. These volume allocation of resources matrices apply to both the with and without commodity tax situations.

Table 20.8: Constant Dollar Domestic Suppy Matrix

	Industry G	**Industry S**	**Industry T**
G	$y_G^{GS}+y_G^{GT}+y_G^{GF}$	0	0
S	0	$y_S^{SG}+y_S^{ST}+y_S^{SF}$	0
T	0	0	$y_T^{TG}+y_T^{TS}+y_T^{TF}$

Table 20.9: Volume Domestic Use Matrix

	Industry G	**Industry S**	**Industry T**
G	0	y_G^{GS}	y_G^{GT}
S	y_S^{SG}	0	y_S^{ST}
T	y_T^{TG}	y_T^{TS}	0

Table 20.10: Volume ROW Supply or Export by Industry and Commodity Matrix

	Industry G	**Industry S**	**Industry T**
G	x_G^{GR}	0	0
S	0	x_S^{SR}	0

[28] If t_{Gm}^{GR} is negative, then the government subsidizes the importation of good G for use by industry G.

T	0	0	x_T^{TR}

Table 20.11: Volume ROW Use or Import by Industry and Commodity Matrix

	Industry G	**Industry S**	**Industry T**
G	m_G^{GR}	m_G^{SR}	m_G^{TR}
S	m_S^{GR}	m_S^{SR}	m_S^{TR}
T	m_T^{GR}	m_T^{SR}	m_T^{TR}

20.95 Comparing the volume allocation of resources matrices defined by Tables 20.8-20.11 with their monetary value at producer price counterparts, it can again be seen that it will generally be impossible to recover the true volume volume or quantity measures along any row by deflating the nominal values by a single price index for that commodity class; i.e., common across industry price deflators will generally not exist. Thus the price statistician's task is a rather daunting one: appropriate specific price deflators or volume extrapolators will in principle be required for *each* nonzero cell in the *System* of nominal value input output matrices in order to recover the correct volume measures.[29]

20.96 As in the previous section, the production sector's nominal value *net supply matrix* that gives the value of net commodity supply by commodity and by industry of origin *at the prices that producers face* can be obtained by adding entries in Tables 20.5 and 20.6 and then subtracting corresponding entries in Tables 20.2 and 20.7. This new net supply matrix gives the value of net commodity supply by commodity and by industry of origin at prices that producers face.

20.97 As in the previous section, the net supply matrix can be aggregated by summing over columns along each row (the resulting value aggregates are the values of *net supply by commodity* at producer prices) or by summing over rows down each column (the resulting value aggregates are equal to *value added by industry* at producer prices).

20.98 The three *value of commodity net supply aggregates at producer prices including taxes and subsidies on output* (the counterparts to the aggregates defined by equations (20.7)-(20.9) turn out to be the following value aggregates:

$$\begin{aligned}(20.20)\quad vf_G \equiv{} & p_G^{GF}y_G^{GF} + p_{Gx}^{GR}x_G^{GR} - p_{Gm}^{GR}m_G^{GR} - p_{Gm}^{SR}m_G^{SR} - p_{Gm}^{TR}m_G^{TR} \\ & - [t_G^{GS}y_G^{GS}+t_G^{GT}y_G^{GT}+t_G^{GF}y_G^{GF}+t_{Gx}^{GR}x_G^{GR}+t_{Gm}^{GR}m_G^{GR}+t_{Gm}^{SR}m_G^{SR}+t_{Gm}^{TR}m_G^{TR}]\ ;\end{aligned}$$

$$\begin{aligned}(20.21)\quad vf_S ={} & p_S^{SF}y_S^{SF} + p_{Sx}^{SR}x_S^{SR} - p_{Sm}^{GR}m_S^{GR} - p_{Sm}^{SR}m_S^{SR} - p_{Sm}^{TR}m_S^{TR} \\ & - [t_S^{SG}y_S^{SG}+t_S^{ST}y_S^{ST}+t_S^{SF}y_S^{SF}+t_{Sx}^{SR}x_S^{SR}+t_{Sm}^{GR}m_S^{GR}+t_{Sm}^{SR}m_S^{SR}+t_{Sm}^{TR}m_S^{TR}]\ ;\end{aligned}$$

$$\begin{aligned}(20.22)\quad vf_T ={} & p_T^{TF}y_S^{SF} + p_{Tx}^{TR}x_S^{TR} - p_{Tm}^{GR}m_T^{GR} - p_{Tm}^{SR}m_T^{SR} - p_{Tm}^{TR}m_T^{TR} \\ & - [p_T^{TG}y_T^{TG}+ p_T^{TS}y_T^{TS}+t_T^{TF}y_S^{SF}+t_{Tx}^{TR}x_S^{TR}+t_{Tm}^{GR}m_T^{GR}+t_{Tm}^{SR}m_T^{SR}+t_{Tm}^{TR}m_T^{TR}].\end{aligned}$$

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[29] Of course, in practice, compromises with the theory will have to be made.

20.99 Looking at equation (20.20), it can be seen that the net value of production of good G at producer prices is equal to $p_G^{GF}y_G^{GF} + p_{Gx}^{GR}x_G^{GR} - p_{Gm}^{GR}m_G^{GR} - p_{Gm}^{SR}m_G^{SR} - p_{Gm}^{TR}m_G^{TR}$, which is the net value of production of commodity G, delivered to the domestic Final Demand and Rest of the World sectors, at final demand prices, less a term in square brackets that represents the net revenue (commodity tax revenue less subsidies for commodity G) that the government sector collects by taxing (or subsidizing) transactions that involve commodity G. The interpretations for vf_S and vf_T are similar.

20.100 The three *industry value added aggregates at producer prices* turn out to be the following value aggregates:

$$(20.23)\ va^G \equiv (p_G^{GS}-t_G^{GS})y_G^{GS}+(p_G^{GT}-t_G^{GT})y_G^{GT}+(p_G^{GF}-t_G^{GF})y_G^{GF} - p_S^{SG}y_S^{SG} - p_T^{TG}y_T^{TG} + (p_{Gx}^{GR}-t_{Gx}^{GR})x_G^{GR} - (p_{Gm}^{GR}+t_{Gm}^{GR})m_G^{GR} - (p_{Sm}^{GR}+t_{Sm}^{GR})m_S^{GR} - (p_{Tm}^{GR}+t_{Tm}^{GR})m_T^{GR};$$

$$(20.24)\ va^S \equiv (p_S^{SG}-t_S^{SG})y_S^{SG}+(p_S^{ST}-t_S^{ST})y_S^{ST}+(p_S^{SF}-t_S^{SF})y_S^{SF} - p_G^{GS}y_G^{GS} - p_T^{TS}y_T^{TS} + (p_{Sx}^{SR}-t_{Sx}^{SR})x_S^{SR} - (p_{Gm}^{SR}+t_{Gm}^{SR})m_G^{SR} - (p_{Sm}^{SR}+t_{Sm}^{SR})m_S^{SR} - (p_{Tm}^{SR}+t_{Tm}^{SR})m_T^{SR};$$

$$(20.25)\ va^T \equiv (p_T^{TG}-t_T^{TG})y_T^{TG}+(p_T^{TS}-t_T^{TS})y_T^{TS}+(p_T^{TF}-t_T^{TF})y_T^{TF} - p_G^{GT}y_G^{GT} - p_S^{ST}y_S^{ST} + (p_{Tx}^{TR}-t_{Tx}^{TR})x_T^{TR} - (p_{Gm}^{TR}+t_{Gm}^{TR})m_G^{TR} - (p_{Sm}^{TR}+t_{Sm}^{TR})m_S^{TR} - (p_{Tm}^{TR}+t_{Tm}^{TR})m_T^{TR}.$$

20.101 Looking at equation (20.23), it can be seen that the *value added produced by Industry G at producer prices*, va^G, is equal to the value of deliveries of good G to Industry S, $(p_G^{GS}-t_G^{GS})y_G^{GS}$,[30] plus the value of deliveries of good G to Industry T, $(p_G^{GT}-t_G^{GT})y_G^{GT}$, plus the value of deliveries of finished goods, less payments to Industry S for service intermediate inputs, $-p_S^{SG}y_S^{SG}$, less payments to Industry T for transportation service intermediate inputs, $-p_T^{TG}y_T^{TG}$, plus the value of exports delivered to the ROW sector, $(p_{Gx}^{GR}-t_{Gx}^{GR})x_G^{GR}$,[31] less payments to the ROW for imports of goods G used by Industry G, $-(p_{Gm}^{GR}+t_{Gm}^{GR})m_G^{GR}$, less payments to the ROW for imports of services S used by Industry G, $-(p_{Sm}^{GR}+t_{Sm}^{GR})m_S^{GR}$, less payments to the ROW for imports of transportation services T used by Industry G, $-(p_{Tm}^{GR}+t_{Tm}^{GR})m_T^{GR}$. The decompositions for the value added produced by Industries S and T, va^S and va^T, are similar.

20.102 Looking at equations (20.20)-(20.22), it can be seen that it is natural to ignore the commodity tax transactions and to sum the remaining transactions involving exports into an aggregate that is the value of exports at final demand prices, $p_{Gx}^{GR}x_G^{GR} + p_{Sx}^{SR}x_S^{SR} + p_{Tx}^{TR}x_S^{TR}$. It is this value aggregate that is equal to the value of X in GDP, valued at final

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[30] Note that Industry S pays Industry G the price p_G^{GS} per unit of good G delivered to Industry S, but Industry G must remit the specific tax t_G^{GS} out of this price to the Government sector.

[31] The foreign demanders for the exports of good G by Industry G pay Industry G the price p_{Gx}^{GR} per unit of the good exported but Industry G must pay out of this amount, the specific export tax t_{Gx}^{GR} to the Government sector, for each unit of good G that is exported.

demand prices. However, looking at the industry value added aggregates defined by equations (20.23)-(20.22), it can be seen that it is natural to work with the net revenues received by the industries for their exports, which are $(p_{Gx}{}^{GR}-t_{Gx}{}^{GR})x_G{}^{GR}$ for Industry G, $(p_{Sx}{}^{SR}-t_{Sx}{}^{SR})x_S{}^{SR}$ for Industry S and $(p_{Tx}{}^{TR}-t_{Tx}{}^{TR})x_T{}^{TR}$ for Industry T. Thus from the viewpoint of industry accounts, it is natural to aggregate these export revenues across industries in order to obtain the value of exports aggregate at producer prices, $(p_{Gx}{}^{GR}-t_{Gx}{}^{GR})x_G{}^{GR} + (p_{Sx}{}^{SR}-t_{Sx}{}^{SR})x_S{}^{SR} + (p_{Tx}{}^{TR}-t_{Tx}{}^{TR})x_T{}^{TR}$. Thus for production accounts that are based on the economic approach to index number theory, it is more appropriate to use tax adjusted producer prices as the pricing concept rather than final demand prices.[32] Similar comments apply to the treatment of imports. Later in this section, it will be shown how final demand based estimates for volume GDP can be reconciled with production based estimates of volume GDP originating in the production sector at producer prices.

20.103 The three final demand value aggregates defined by (20.20)-(20.22) can be summed and the resulting value aggregate is the GDP_P generated by the economy's production sector at producer prices. Note that we have added the subscript P to this GDP concept at producer prices to distinguish it from the more traditional concept of GDP at final demand prices, which we denote by GDP_F. The two GDP concepts will be reconciled later.

20.104 The value added aggregates at producer prices defined by (20.23)-(20.25) can also be summed and this sum will also equal GDP_P since the two methods for forming estimates of GDP_P are simply alternative methods for summing over the elements of the net supply matrix.[33] Thus the following equation must hold:

$$(20.26)\ GDP_P \equiv vf_G + vf_S + vf_T = va^G + va^S + va^T.$$

20.105 It is useful to explicitly write out GDP_P as the sum of the three final demand aggregates defined (20.23)-(20.25). After some rearrangement of terms the following equation is obtained:

$$\begin{aligned}(20.27)\ GDP_P &= vf_G + vf_S + vf_T \\ &= GDP_F - T \\ &= [C+I+G] + X - M - T\end{aligned}$$

where T is the *value of commodity tax net revenues* (taxes less subsidies) defined as the sum of the following terms

$$\begin{aligned}(20.28)\ T \equiv\ & t_G{}^{GS}y_G{}^{GS}+t_G{}^{GT}y_G{}^{GT}+t_G{}^{GF}y_G{}^{GF}+t_{Gx}{}^{GR}x_G{}^{GR}+t_{Gm}{}^{GR}m_G{}^{GR}+t_{Gm}{}^{SR}m_G{}^{SR}+t_{Gm}{}^{TR}m_G{}^{TR} \\ & + t_S{}^{SG}y_S{}^{SG}+t_S{}^{ST}y_S{}^{ST}+t_S{}^{SF}y_S{}^{SF}+t_{Sx}{}^{SR}x_S{}^{SR}+t_{Sm}{}^{GR}m_S{}^{GR}+t_{Sm}{}^{SR}m_S{}^{SR}+t_{Sm}{}^{TR}m_S{}^{TR}\end{aligned}$$

[32] It is also appropriate to use these tax adjusted producer prices when constructing a Producer Price Index which is based on the economic approach to index number theory.

[33] The first method sums entries along rows first and then sums down the sum column whereas the second method sums entries down columns first and then sums across the sum row.

$$+ p_T^{TG}y_T^{TG} + p_T^{TS}y_T^{TS} + t_T^{TF}y_S^{SF} + t_{Tx}^{TR}x_S^{TR} + t_{Tm}^{GR}m_T^{GR} + t_{Tm}^{SR}m_T^{SR} + t_{Tm}^{TR}m_T^{TR} ;$$

and GDP_F is *the value of GDP at final demand prices* defined as the sum of the following components of final demand at final demand prices:

$$(20.29)\ GDP_F \equiv [p_G^{GF}y_G^{GF} + p_S^{SF}y_S^{SF} + p_T^{TF}y_S^{SF}] + [p_{Gx}^{GR}x_G^{GR} + p_{Sx}^{SR}x_S^{SR} + p_{Tx}^{TR}x_S^{TR}]$$
$$- [p_{Gm}^{GR}m_G^{GR} + p_{Gm}^{SR}m_G^{SR} + p_{Gm}^{TR}m_G^{TR} + p_{Sm}^{GR}m_S^{GR} + p_{Sm}^{SR}m_S^{SR}$$
$$+ p_{Sm}^{TR}m_S^{TR} + p_{Tm}^{GR}m_T^{GR} + p_{Tm}^{SR}m_T^{SR} + p_{Tm}^{TR}m_T^{TR}]$$
$$= [C+G+I] + [X] - [M].$$

20.106 Note that the 15 terms that do not involve taxes on the right hand side of (20.27), which define GDP_F, correspond to the 15 terms on the right hand side of equation (20.11), which provided our initial decomposition of GDP when there were no commodity taxes. However, when there are commodity taxes (and commodity subsidies), the new decomposition of GDP_P requires that the 21 tax terms defined by (20.28) be subtracted from the right hand side of (20.27). Note that using definition (20.29), the identity (20.27) can be rewritten in the following form:

$$(20.30)\ GDP_F = GDP_P + T:$$

20.107 Thus the value of production at final demand prices, GDP_F, is equal to the value of production at producer prices, GDP_P, plus commodity tax revenues less commodity tax subsidies, T, which is a traditional national income accounting identity.

20.108 As was discussed in the previous section, there are three ways that can be used to construct a volume or quantity index of net outputs (at producer prices) produced by the production sector:

- Sum the two Supply matrices and subtract the two Use matrices and look at the cell entries in the resulting matrix. Collecting up all of the nonzero transactions, it can be seen that there are 48 distinct price times quantity transactions. If there is a negative sign associated with any one of these terms, that negative sign is attached to the quantity. Now apply normal index number theory to these 48 price times quantity components of the aggregate.
- Sum up the value added aggregates defined by (20.23)-(20.25). The resulting value added aggregate will have the same 48 separate price times quantity components that occurred in the first method of aggregation. If a value component has a negative sign associated with it, then attach the negative sign to the quantity (so that all prices will always be positive). Now apply normal index number theory to these 48 price times quantity components of the aggregate. This method will generate the same results as the first method listed above.
- Sum up the final demand value aggregates defined by (20.20)-(20.22). The resulting value of final demand aggregate will have 36 separate price times quantity components. If a value component has a negative sign associated with it, then attach

the negative sign to the quantity. Now apply normal index number theory to these 36 price times quantity components of the aggregate.

20.76 It is evident that the quantity index or the volume estimate for GDP will be the same using methods 1 and 2 listed above since the two methods generate exactly the same set of 48 separate price times quantity components in the value aggregate. However, it is not evident that volume estimates for GDP based on method 3 will coincide with those generated using methods 1 and 2 since there are 48 price times quantity components to be aggregated when we use methods 1 or 2 compared to only 36 components when we use method 3. However, equations (20.12)-(20.19) in the previous section (with some obvious changes in notation) continue to hold in this new framework with commodity taxes and subsidies. Thus value added (at producer prices) Laspeyres, Paasche and Fisher quantity indexes will be equal to their final demand counterparts, where the 21 terms involving taxes are used in the formulae. Note that the specific tax terms play the role of prices in these index number formulae and the associated quantities have negative signs attached to them when calculating these final demand (at producer prices) index numbers.

20.77 The equality (20.19) between the two methods for constructing an aggregate volume index for GDP using the Fisher quantity index as the index number formula can be extended to the case where the implicit Törnqvist quantity index is used as the index number formula. In this case, the value aggregates are deflated by the Törnqvist price index and by writing out the formulae, it is straightforward to show that $P_T^{va}(p^{va0},p^{va1},q^{va0},q^{va1})$, the Törnqvist price index using the 48 price times quantity components in the value added aggregate, is equal to $P_T^{fd}(p^{fd0},p^{fd1},q^{fd0},q^{fd1})$, the Törnqvist price index using the 36 price times quantity components in the final demand aggregate.[34]

20.78 As noted in the previous section, GDP_P can be calculated using two stage aggregation where the first stage calculates volume value added (at producer prices) by industry. The two stage estimates of GDP_P will coincide exactly with their single stage counterparts if the Laspeyres or Paasche formulae are used and will approximately coincide if the Fisher formula is used. It should be noted that the value added at producer prices approach for the calculation of industry aggregates is suitable for productivity analysis purposes.[35] It should be emphasized that in order to construct accurate productivity statistics for each industry, it

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[34] However, in order to obtain this equality for the Törnqvist price index, it is necessary to treat each indirect tax as a separate price component for both the value added and final demand methods of aggregation; i.e., if the terms involving final demand prices and taxes are combined into single producer prices and then fed into the Törnqvist price index formula when using the value added approach, then the resulting index value is not necessarily equal to the Törnqvist price index that directly aggregates the 36 components of final demand. Thus in Chapter 18, since the second method of aggregation was used, the exact equality of the two Törnqvist price indices did not hold.

[35] See Jorgenson and Griliches (1967) (1972) for an early exposition of how productivity accounts could be set up. The indirect tax conventions used in this chapter are consistent with the recommendations of Jorgenson and Griliches (1972; 85) on the treatment of indirect taxes in a set of productivity accounts.

generally will be necessary to construct *separate price deflators for each nonzero cell in the augmented input output tables* that have been suggested in this chapter.

20.79 The final topic for this section is how to reconcile volume estimates for GDP at final demand prices, GDP_F, with volume estimates for GDP at producer prices, GDP_P. Recall equation (20.30), which said that GDP_F equals GDP_P plus T. Suppose that data are available for two periods which respect equation (20.30) in each period and a quantity index is constructed GDP_F defined by (20.29) with 15 separate price times quantity components. Then noting that GDP_P is defined by the sum of (20.23)-(20.25) with 48 price times quantity components,[36] and T is defined by (20.28) with 21 price times quantity components, we could combine these transactions and construct an alternative quantity index for this sum of GDP_P and T value aggregate using the same index number formula. Using the same method of proof as was used in the previous section, it can be shown that the resulting volume estimates for GDP_F and $GDP_P + T$ will coincide if the Laspeyres, Paasche or Fisher formulae are used. For the $GDP_P + T$ aggregate, two stage aggregation could be used where the first stage value aggregates are GDP_P, GDP at producer prices, and T, commodity tax revenue less commodity subsidies. The two stage estimates will be exactly equal to the corresponding single stage estimates if the Laspeyres or Paasche formulae are used for the quantity index and will be approximately equal if the Fisher formula is used. This type of decomposition will enable analysts to relate volume growth in final demand GDP_F to volume growth in GDP_P at producer prices plus commodity tax effects. More generally, the identity (20.30) can be used to estimate GDP_F if the statistical agency is able to estimate GDP_P and in addition, the statistical agency can form estimates of the 21 tax times quantity terms on the right hand side of (20.28).[37]

C. The Artificial Data Set

C.1 The Artificial Data Set Framework: Real Supply and Use Matrices

20.80 An artificial data set is presented in this section for the Supply and Use Tables outlined in the previous section. It is useful to expand the commodity classification from one good G to four goods, G1, G2, G3 and G4, and from one service to two services, S1 and S2. The four goods are:

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[36] Alternatively, the tax terms could be combined with the final demand prices and then there would only be 27 price times quantity value transactions in the aggregate.

[37] Conversely, the identity $GDP_P = GDP_F - T$ implies that if the statistical agency is able to estimate GDP_F, and in addition, the statistical agency can make estimates of the 21 tax times quantity terms on the right hand side of (20.28), then estimates of GDP_P can be made. Thus the allocation of commodity taxes and subsidies to the correct cells of the system of Supply and Use matrices is important. These observations on the importance of commodity taxes in the Input Output framework are generalizations of observations made by Diewert (2006; 303-304) in the context of a model of a closed economy.

- G1, agricultural products or food good;
- G2, crude oil or more generally, energy products;
- G3, an imported pure intermediate good that is used by the domestic goods producing industry and
- G4, a general consumption non-energy, non-food good.

The two services are:

- S1, traditional services and
- S2, high technology services such as telecommunications and internet access.

The remaining commodity in the commodity classification is T, transportation services.

20.81 The constant dollar table counterparts to Tables 20.8-20.11 are now modified into Tables 20.12-20.15 below. The counterpart to Table 20.8 is Table 20.12. This matrix shows the production by commodity and by industry that is delivered to domestic demanders. Thus y_{G4}^{GS} denotes the quantity of good G4 that is delivered by the goods producing industry G to the services industry S, y_{G4}^{GT} denotes the quantity of good G4 that is delivered by the goods producing industry G to the transportation industry T, y_{G4}^{GF} denotes the quantity of good G4 that is delivered by the goods producing industry G to the domestic final demand sector F, y_{G1}^{SF} denotes the quantity of good G1 (food imports) delivered by the services industry S (which includes retailing and wholesaling) to the domestic final demand sector F, y_{G2}^{SF} denotes the quantity of good G2 (energy imports) delivered by the services industry S (which includes retailing and wholesaling) to the domestic final demand sector F, y_{S1}^{SG} denotes the quantity of traditional services S1 that is delivered by the services industry S to the goods producing industry G, y_{S2}^{SG} denotes the quantity of high tech services S2 that is delivered by the services industry S to the goods producing industry G, y_{T}^{TG} denotes the quantity of transportation services T that is delivered by the transportation industry T to the goods producing industry G, and so on.

Table 20.12: Real Domestic Supply Matrix

	Industry G	**Industry S**	**Industry T**
G1	0	y_{G1}^{SF}	0
G2	0	y_{G2}^{SF}	0
G3	0	0	0
G4	$y_{G4}^{GS}+y_{G4}^{GT}+y_{G4}^{GF}$	0	0
S1	0	$y_{S1}^{SG}+y_{S1}^{ST}+y_{S1}^{SF}$	0
S2	0	$y_{S2}^{SG}+y_{S2}^{ST}+y_{S2}^{SF}$	0
T	0	0	$y_{T}^{TG}+y_{T}^{TS}+y_{T}^{TF}$

20.82 Looking at the entries in Table 20.13, it can be seen that there is no domestic production of goods G1 (agricultural products) and G2 (crude oil) by Industries G and T and no domestic production of G3 (the imported intermediate good used by the goods producing industry G) by any of the industries. Industry G produces good G4 and delivers y_{G4}^{GS} units of this good to the service industry S to be used as an intermediate input there, delivers y_{G4}^{GT}

units of this good to the transportation industry T to be used as an intermediate input there and delivers $y_{G4}{}^{GF}$ units of this good to the domestic final demand sector F. Similarly, Industry S produces the general service commodity S1 and delivers $y_{S1}{}^{SG}$ units of this commodity to the goods producing industry G to be used as an intermediate input there, delivers $y_{S1}{}^{ST}$ units of this service to the transportation industry T to be used as an intermediate input there and delivers $y_{S1}{}^{SF}$ units of this service to the domestic final demand sector F. Industry S also produces the high technology service commodity S2 and delivers $y_{S2}{}^{SG}$ units of this commodity to the goods producing industry G to be used as an intermediate input there, delivers $y_{S2}{}^{ST}$ units of this service to the transportation industry T to be used as an intermediate input there and delivers $y_{S2}{}^{SF}$ units of this service to the domestic final demand sector F. It is also assumed that the service industry imports G1 (agricultural produce) and G2 (crude oil) and stores and distributes these imports to the household sector; these are the deliveries $y_{SG1}{}^{SF}$ and $y_{SG2}{}^{SF}$.[38] Finally, Industry T produces the transportation services commodity T and delivers $y_T{}^{TG}$ units of this commodity to the goods producing industry G to be used as an intermediate input there, delivers $y_T{}^{TS}$ units of these transport services to the service industry S to be used as an intermediate input there and delivers $y_T{}^{TF}$ units of transport services directly to the domestic final demand sector F.

20.83 The counterpart to Table 20.9 is now Table 20.13. This matrix lists the industry demands for commodities that originate from domestic sources; i.e., it shows the industry by commodity intermediate input demands for commodities that are supplied from domestic sources.

Table 20.13: Real Domestic Use Matrix

	Industry G	**Industry S**	**Industry T**
G1	0	0	0
G2	0	0	0
G3	0	0	0
G4	0	$y_{G4}{}^{GS}$	$y_{G4}{}^{GT}$
S1	$y_{S1}{}^{SG}$	0	$y_{S1}{}^{ST}$
S2	$y_{S2}{}^{SG}$	0	$y_{S2}{}^{ST}$
T	$y_T{}^{TG}$	$y_T{}^{TS}$	0

20.84 Since there is no domestic production of goods G1-G3, the rows that correspond to these commodities in Table 20.13 all have 0 entries. The remainder of the Table is the same as Table 20.9. Note that the domestic intersectoral transfers of goods and services in Tables

[38] This is not the only way the accounts could be set up. Note that the distribution services (in distributing G1 and G2) that the domestic service industry provides in this accounting framework is on a gross basis whereas the treatment of transportation services in Industry T is on a net basis; i.e., the present setup treats transportation services as a margin industry whereas the services associated with the direct distribution of imports to households is not treated in this way. This treatment of imports makes reconciliation of the production accounts with the final demand accounts fairly straightforward.

20.12 and 20.13 match up exactly; i.e., the 8 nonzero quantities in Table 20.13 are exactly equal to the corresponding entries in Table 20.12.

The counterpart to Table 20.10 is now Table 20.14.

Table 20.14: Real ROW Supply or Export by Industry and Commodity Matrix

	Industry G	**Industry S**	**Industry T**
G1	0	0	0
G2	0	0	0
G3	0	0	0
G4	$x_{G4}{}^{GR}$	0	0
S1	0	$x_{S1}{}^{SR}$	0
S2	0	0	0
T	0	0	$x_T{}^{TR}$

20.85 Since there is no exportation of goods G1-G3, the rows that correspond to these commodities in Table 20.14 all have 0 entries. The remainder of the Table is the same as Table 20.10. Thus Industry G exports $x_{G4}{}^{GR}$ units of Good G4, Industry S exports $x_{S1}{}^{SR}$ units of traditional services S1 and no units of high tech services and Industry T exports $x_T{}^{TR}$ units of transportation services to the Rest of the World.

20.86 The counterpart to Table 20.11 is now Table 20.15. This matrix lists the industry demands for commodities that originate from foreign sources; i.e., it shows the industry by commodity intermediate input demands for intermediate inputs from foreign sources.

Table 20.15: Real ROW Use or Import by Industry and Commodity Matrix

	Industry G	**Industry S**	**Industry T**
G1	$m_{G1}{}^{GR}$	$m_{G1}{}^{SR}$	0
G2	$m_{G2}{}^{GR}$	$m_{G2}{}^{SR}$	$m_{G2}{}^{TR}$
G3	$m_{G3}{}^{GR}$	0	0
G4	0	0	0
S1	$m_{S1}{}^{GR}$	$m_{S1}{}^{SR}$	0
S2	0	0	0
T	0	$m_T{}^{SR}$	$m_T{}^{TR}$

20.87 From Table 20.15, it can be seen that the goods producing industry uses $m_{G1}{}^{GR}$ units of agricultural imports, $m_{G2}{}^{GR}$ units of crude oil imports, $m_{G3}{}^{GR}$ units of a pure imported intermediate good and $m_{S1}{}^{GR}$ units of imported service inputs. Industry G does not import the domestically produced good, G4, nor does it import transportation services in this simplified example. Industry S imports $m_{G1}{}^{SR}$ units of agricultural goods (for distribution to domestic households), $m_{G2}{}^{sR}$ units of crude oil (for distribution to households and own use), $m_{S1}{}^{GR}$ units of foreign general services and $m_T{}^{SR}$ units of foreign transportation services. Industry T imports $m_{G2}{}^{TR}$ units of crude oil and $m_T{}^{TR}$ units of foreign sourced transportation services.

C.2 The Artificial Data Set Framework: Value Supply and Use Matrices

20.88 The value matrix counterparts to the two Supply and two Use matrices listed in section B.1 above will now be listed in the present section. Table 20.16 listed below is the counterpart to Table 20.5.

Table 20.16: Nominal Value Domestic Supply Matrix with Commodity Taxes

Industry G	Industry S	Industry T
0	$(p_{G1}^{SF}-t_{G1}^{SF})y_{G1}^{SF}$	0
0	$(p_{G2}^{SF}-t_{G2}^{SF})y_{G2}^{SF}$	0
0	0	0
$(p_{G4}^{GS}-t_{G4}^{GS})y_{G4}^{GS}$ $+(p_{G4}^{GT}-t_{G4}^{GT})y_{G4}^{GT}$ $+(p_{G4}^{GF}-t_{G4}^{GF})y_{G4}^{GF}$	0	0
0	$(p_{S1}^{SG}-t_{S1}^{SG})y_{S1}^{SG}$ $+(p_{S1}^{ST}-t_{S1}^{ST})y_{S1}^{ST}$ $+(p_{S1}^{SF}-t_{S1}^{SF})y_{S1}^{SF}$	0
0	$(p_{S2}^{SG}-t_{S2}^{SG})y_{S2}^{SG}$ $+(p_{S2}^{ST}-t_{S2}^{ST})y_{S2}^{ST}$ $+(p_{S2}^{SF}-t_{S2}^{SF})y_{S2}^{SF}$	0
0	0	$(p_T^{TG}-t_T^{TG})y_T^{TG}$ $+(p_T^{TS}-t_T^{TS})y_T^{TS}$ $+(p_T^{TF}-t_T^{TF})y_T^{TF}$

20.89 All of the prices which begin with the letter p are the prices that *domestic final demanders* pay for a unit of the commodity (except for minor complications with respect to the treatment of export prices). In the above Table, these prices correspond to *purchasers' prices* in the *System of National Accounts 1993*.[39] However, the industry sellers of these commodities do not generally receive the full final demand price: commodity taxes less commodity subsidies must be subtracted from these final demand prices in order to obtain the net prices that are listed in the above Table. These *net selling prices* are the prices that the industrial producers actually receive for their sales of outputs to domestic demanders. In the

[39] See part (a) of paragraph 15.28 in *SNA 1993*.

above Table, these prices correspond to *basic prices* in *SNA 1993*.[40] The notation used for prices in Table 20.16 matches the notation used for quantities in Table 20.12.

20.90 The reader should note that in this chapter, for convenience, the p prices will be referred to as *final demand prices* and the p−t prices will be referred to as *producer prices*. Conceptually, the *final demand prices* are the prices that domestic final demanders pay per unit for their purchases of commodities delivered to final demand categories. However, for an exported commodity, the final demand price is not the total purchase price (including transportation services provided by foreign establishments, import duties and other applicable commodity taxes) that the foreign importer pays for the commodity; rather, in this case, the final demand price is only the payment made to the domestic producer by the foreign importer. Conceptually, *producer prices* are the prices that domestic producers receive per unit of output produced that is sold or the prices that domestic producers pay per unit of input that is purchased (including applicable commodity taxes and all transportation margins).[41]

Table 20.17 listed below is the counterpart to Table 20.2. It is also the value counterpart to Table 20.13 listed above.

Table 20.17: Nominal Value Domestic Use Matrix

	Industry G	**Industry S**	**Industry T**
G1	0	0	0
G2	0	0	0
G3	0	0	0
G4	0	$p_{G4}^{GS}y_{G4}^{GS}$	$p_{G4}^{GT}y_{G4}^{GT}$
S1	$p_{S1}^{SG}y_{S1}^{SG}$	0	$p_{S1}^{ST}y_{S1}^{ST}$
S2	$p_{S2}^{SG}y_{S2}^{SG}$	0	$p_{S2}^{ST}y_{S2}^{ST}$
T	$p_{T}^{TG}y_{T}^{TG}$	$p_{T}^{TS}y_{T}^{TS}$	0

20.91 Note that in Table 20.16, Industry G receives only the revenue $(p_{G4}^{GS}-t_{G4}^{GS})y_{G4}^{GS}$ for its sales of Commodity G4 to Industry S, whereas in Table 20.17, Industry S pays the amount $p_{G4}^{GS}y_{G4}^{GS}$ for these purchases of intermediate inputs from Industry G. The difference between these two intersectoral value flows is $t_{G4}^{GS}y_{G4}^{GS}$, the tax (less subsidy) payments

[40] See paragraphs 15.28-15.33 in *SNA 1993*. Note that the tax terms in Tables 20.5-20.8 are equal to per unit (or specific) commodity taxes less per unit commodity subsidies. These two effects could be distinguished separately at the cost of additional notational complexity.

[41] If the production accounts are to be used in order to measure Total Factor Productivity growth using the economic approach suggested by Jorgenson and Griliches (1967) (1972), it is important to use the prices that producers face in the accounting framework. The treatment of commodity taxes suggested in this Manual is consistent with the treatment suggested by Jorgenson and Griliches who advocated the following treatment of indirect taxes: "In our original estimates, we used gross product at market prices; we now employ gross product from the producers' point of view, which includes indirect taxes levied on factor outlay, but excludes indirect taxes levied on output." Dale W. Jorgenson and Zvi Griliches (1972; 85).

made by Industry G to the government on this intersectoral value flow. Thus the values of domestic intersectoral transfers of goods and services in Tables 20.16 and 20.17 do not match up exactly unless the commodity tax less subsidy terms t_{S1}^{SG}, t_{G4}^{GS} and so on are all zero.

20.92 The counterpart to Table 20.6 is now Table 20.18, which in turn is the value counterpart to the real Table 20.14.

20.93 Table 20.18: Value ROW Supply or Export by Industry and Commodity Matrix

	Industry G	Industry S	Industry T
G1	0	0	0
G2	0	0	0
G3	0	0	0
G4	$(p_{G4x}^{GR}-t_{G4x}^{GR})x_{G4}^{GR}$	0	0
S1	0	$p_{S1x}^{SR}x_{S1}^{SR}$	0
S2	0	0	0
T	0	0	$p_{Tx}^{TR}x_T^{TR}$

20.94 Since there is no exportation of goods G1-G3, the rows that correspond to these commodities in Table 20.18 all have 0 entries. The remainder of the Table is straightforward. Thus Industry G exports x_{G4}^{GR} units of Good G4,, the foreign final demander pays the price p_{G4x}^{GR} per unit but the exporting industry gets only the amount $p_{G4x}^{GR}-t_{G4x}^{GR}$ per unit; i.e., the government gets the per unit (net) revenue t_{G4x}^{GR} on these sales if it imposes a (net) export tax equal to t_{G4x}^{GR}. Similarly, net export taxes (if applicable) must be subtracted from the final demand prices for the other industries. In the numerical example which follows, it will be assumed that the net export tax in Industry G is negative (so that exports are subsidized in industry G) and it will be assumed that taxes in Industries S and T are zero.

The counterpart to Table 20.7 is now Table 20.19, which in turn is the value counterpart to the real Table 20.15.

Table 20.19: Value ROW Use or Import by Industry and Commodity Matrix

	Industry G	Industry S	Industry T
G1	$(p_{G1m}^{GR}+t_{G1m}^{GR})m_{G1}^{GR}$	$(p_{G1m}^{SR}+t_{G1m}^{SR})m_{G1}^{SR}$	0
G2	$(p_{G2m}^{GR}+t_{G2m}^{GR})m_{G2}^{GR}$	$(p_{G2m}^{SR}+t_{G2m}^{SR})m_{G2}^{SR}$	$(p_{G2m}^{TR}+t_{G2m}^{TR})m_{G2}^{TR}$
G3	$(p_{G3m}^{GR}+t_{G3m}^{GR})m_{G3}^{GR}$	0	0
G4	0	0	0
S1	$(p_{S1m}^{GR}+t_{S1m}^{GR})m_{S1}^{GR}$	$(p_{S1m}^{SR}+t_{S1m}^{SR})m_{S1}^{SR}$	0
S2	0	0	0
T	0	$(p_{Tm}^{SR}+t_{Tm}^{SR})m_T^{SR}$	$(p_{Tm}^{TR}+t_{Tm}^{TR})m_T^{TR}$

20.95 It should be straightforward for the reader to interpret the final demand prices (these terms begin with p) and the accompanying import duties, excise duties and other commodity taxes on imports (these terms begin with t). The quantities of imports (these terms begin with an m) are the same as the quantity terms in the corresponding real table, Table 20.15. From a practical point of view, governments have a tendency to tax imports (so that the tax terms in this table will tend to be positive) and to subsidize exports (so that the tax terms in the previous table will tend to be 0 or negative).

B.4 Industry G Prices and Quantities

20.96 All of the price and quantity series that will be used in this chapter are listed in the four nominal value Supply and Use matrices that are listed in Tables 20.16-20.19. The eleven final demand price series that form part of the Industry G data in these matrices are listed for 5 periods in Table 20.20. The commodity that the price refers to is listed in the first row of the Table.

Table 20.20 Industry G Final Demand Prices for All Transactions

	G4	G4	G4	S1	S2	T	G4	G1	G2	G3	S1
	p_{G4}^{GS}	p_{G4}^{GT}	p_{G4}^{GF}	p_{S1}^{SG}	p_{S2}^{SG}	p_T^{TG}	p_{G4x}^{GR}	p_{G1m}^{GR}	p_{G2m}^{GR}	p_{G3m}^{GR}	p_{S1m}^{GR}
1	0.9	0.9	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0
2	1.2	1.2	1.3	1.5	0.8	1.6	1.3	1.4	2.0	0.8	1.4
3	1.5	1.5	1.6	1.8	0.6	1.5	1.6	0.9	1.5	0.6	1.7
4	1.55	1.55	1.65	1.9	0.4	1.3	1.5	1.3	0.9	0.4	1.8
5	1.6	1.6	1.7	2.0	0.2	1.8	1.4	1.5	2.1	0.3	1.9

20.97 Some points to note about the price entries in Table 20.20 are as follows. The Industry G final demand prices that it faces for deliveries of Commodity G4 to the service industry, p_{G4}^{GS}, to the transportation services industry, p_{G4}^{GT}, and for exports, p_{G4x}^{GR}, are all much the same: prices trend up fairly rapidly for periods 2 and 3 and then level off for periods 4 and 5. However, the final demand price for deliveries of G4 to the domestic final demand sector F, p_{G4}^{GF}, is somewhat higher than the corresponding prices for deliveries of G4 to the service sector S, p_{G4}^{GS}, and to the transportation sector T, p_{G4}^{GT}, due to higher commodity taxes on deliveries to sector F. The price of traditional domestic services used as an intermediate input by Industry G, p_{S1}^{SG}, also increases rapidly initially and then levels off for the last two periods. However, the price of high tech domestic services used as an intermediate input by Industry G, p_{S2}^{SG}, drops rapidly throughout the sample period. The price of transportation services used as an intermediate input by Industry G, p_T^{TG}, increases dramatically in period 2 due to the increase the price of imported oil and then decreases for the next two periods as the price of oil drops before increasing again in period 5. The price of agricultural imports into Industry G, p_{G1m}^{GR}, fluctuates considerably from period to period but overall, agricultural import prices do not increase as rapidly as many other prices. The price of oil imports into Industry G, p_{G2m}^{GR}, fluctuates violently, doubling in period 2, then falling so that by period 4, the price is below the period 1 price but then the price more than doubles for period 5. The price of the imported intermediate good, p_{G3m}^{GR}, steadily drops at a rapid pace over the 5 periods.[42] Finally, the price of the imported services commodity, p_{Sm}^{GR}, increases rapidly over periods 2 and 3 but then the rate of price increase slows down. Over the entire period, the price of services tends to increase somewhat more rapidly than the price of manufactured output, G4.

20.98 The eleven commodity tax series that form part of the Industry G taxes listed in Tables 20.16-20.19 are listed for 5 periods in Table 20.21. Recall that by convention, the selling industry pays all commodity taxes so the taxes on Industry G's purchases of intermediate inputs from Industries S and T, t_{S1}^{SG}, t_{S2}^{SG} and t_T^{TG}, are all identically equal to zero. However, in Table 20.21, these tax rates are listed (with 0 entries) so that Table 20.21 is dimensionally comparable to Table 20.20.

Table 20.21 Industry G Commodity Taxes

	G4	G4	G4	S1	S2	T	G4	G1	G2	G3	S1
Period	t_{G4}^{GS}	t_{G4}^{GT}	t_{G4}^{GF}	t_{S1}^{SG}	t_{S2}^{SG}	t_T^{TG}	t_{G4x}^{GR}	t_{G1m}^{GR}	t_{G2m}^{GR}	t_{G3m}^{GR}	t_{S1m}^{GR}
1	0.05	0.05	0.10	0	0	0	−0.10	0.11	0.15	0.10	0.05
2	0.07	0.07	0.15	0	0	0	−0.13	0.14	0.20	0.08	0.07
3	0.08	0.08	0.20	0	0	0	−0.16	0.09	0.25	0.06	0.08
4	0.08	0.08	0.22	0	0	0	−0.15	0.13	0.20	0.04	0.09
5	0.09	0.09	0.23	0	0	0	−0.05	0.15	0.25	0.03	0.10

20.99 Note that the taxes listed above are all positive or zero except that the exports of good G4 by Industry G are subsidized so the taxes t_{G4x}^{GR} have a negative sign attached to them instead of the usual positive sign.

20.100 The eleven quantity series that form part of the Industry G data in Tables 20.16-20.19 are listed for 5 periods in Table 20.22.

Table 20.22 Industry G Quantities of Outputs and Intermediate Inputs

	G4	G4	G4	S1	S2	T	G4	G1	G2	G3	S1
	y_{G4}^{GS}	y_{G4}^{GT}	y_{G4}^{GF}	y_{S1}^{SG}	y_{S2}^{SG}	y_T^{TG}	x_{G4x}^{GR}	m_{G1m}^{GR}	m_{G2m}^{GR}	m_{G3m}^{GR}	m_{S1m}^{GR}
1	5	2	35	4	2	3	25	5	10	10	2
2	6	2.5	40	5	4	3.5	28	6	12	13	2
3	7	3	45	6	8	4	32	7	15	19	3
4	7	3.5	49	8	14	5	40	7.5	18	25	4
5	8	4	55	10	20	6	54	8	15	35	6

20.101 The quantities of good G4 produced by Industry G, y_{G4}^{GS}, y_{G4}^{GT}, y_{G4}^{GF}, which are deliveries to the domestic services industry, the domestic transportation industry and the

[42] Think of this pure imported intermediate as being high tech equipment, which has been dropping in price due to the computer chip revolution.

domestic final demand sector respectively, all grow at roughly the same rate. However, the quantities of G4 exported by Industry G, $x_{G4x}{}^{GR}$, grow a bit more rapidly, particularly during the final two periods. The quantity of traditional domestic services used as an intermediate input by Industry G, $y_{S1}{}^{SG}$, more than doubles over the 5 periods but the quantity of high tech services used as an intermediate input, $y_{S1}{}^{SG}$, grows tenfold due to the rapid price drop in this commodity. The quantity of domestic transportation services used as an intermediate input by Industry G, $y_T{}^{TG}$, exactly doubles over the 5 periods. The quantity of agricultural imports used by Industry G, $m_{G1m}{}^{GR}$, increases steadily from 5 units to 8 units while the quantity of oil imports increases from 10 in period 1 to 15 in period 3 but then the growth rate slows over the final two periods. Imports of the high technology pure intermediate imported good, $m_{G3m}{}^{GR}$, increase rapidly from 10 to 35 units, reflecting the real world tendency towards globalization and international outsourcing. Finally, imports of service inputs into Industry G increase rapidly, growing from 2 units in period 1 to 6 units in period 5.

B.5 Industry S Prices and Quantities

20.102 The fifteen final demand price series that form part of the Industry S data in Tables 20.16-20.19 are listed for 5 periods in Table 20.20.

Table 20.23 Industry S Final Demand Prices

	G1	G2	S1	S1	S1	S2	S2	S2
Period	$p_{G1}{}^{SF}$	$p_{G2}{}^{SF}$	$p_{S1}{}^{SG}$	$p_{S1}{}^{ST}$	$p_{S1}{}^{SF}$	$p_{S2}{}^{SG}$	$p_{S2}{}^{ST}$	$p_{S2}{}^{SF}$
1	1.2	1.4	1.0	1.0	1.3	1.0	1.0	1.15
2	1.5	2.8	1.5	1.4	1.8	0.8	0.8	0.94
3	1.2	2.2	1.8	1.7	2.2	0.6	0.6	0.72
4	1.6	1.5	1.9	1.8	2.4	0.4	0.4	0.45
5	1.7	3.0	2.0	1.9	2.6	0.2	0.2	0.23

	G4	T	S1	G1	G2	S1	T
Period	$p_{G4}{}^{GS}$	$p_T{}^{TS}$	$p_{S1x}{}^{SR}$	$p_{G1m}{}^{SR}$	$p_{G2m}{}^{SR}$	$p_{S1m}{}^{SR}$	$p_{Tm}{}^{SR}$
1	0.9	1.0	1.0	1.0	1.0	1.0	1.0
2	1.2	1.6	1.3	1.3	2.1	1.3	1.6
3	1.5	1.5	1.6	1.0	1.6	1.6	1.5
4	1.55	1.3	1.5	1.4	1.1	1.7	1.3
5	1.6	1.8	1.4	1.5	2.2	1.7	1.8

20.103 Some points to note about the price entries in Table 20.23 are as follows. The prices of service sector deliveries to Industry G, $p_{S1}{}^{SG}$ and $p_{S2}{}^{SG}$, and the prices of deliveries of good G4 from Industry G to Industry S, $p_{G4}{}^{GS}$, are exactly the same as in Tables 20.20 and 20.23. This reflects the bilateral nature of transactions between sectors. Industry S sells Commodities S1 and S2 to Industries G and T (these are the prices $p_{S1}{}^{SG}$ and $p_{S2}{}^{SG}$ for sales to Industry G and $p_{S1}{}^{ST}$ and $p_{S2}{}^{ST}$ for sales to Industry T) and it sells commodities S1 and S2 to the domestic Final Demand sector F at prices $p_{S1}{}^{SF}$ and $p_{S2}{}^{SF}$ and it sells S1 to the Rest of the World R as an export at the price $p_{S1x}{}^{SR}$. The Industry S final demand selling prices are much the same over these 4 destinations, except that export price for S1 falls off somewhat and the selling prices to the domestic Final Demand Sector for the high technology service S2 are somewhat higher, reflecting a higher level of final demand taxation. Industry S also imports G1 (agricultural or food imports for resale to domestic households), G2 (oil imports for resale to domestic households) and it also imports some foreign general services S1 and some foreign transportation services T. These import prices are $p_{G1m}{}^{SR}$, $p_{G2m}{}^{SR}$, $p_{S1m}{}^{SR}$ and $p_{Tm}{}^{SR}$ respectively. The import prices for these first 3 classes of imports are much the same as the corresponding import prices that applied to the imports of these commodities by Industry G. The price of imported transportation services, $p_{Tm}{}^{SR}$, is the same as the price of domestic transportation services provided to Industry S, $p_T{}^{TS}$. Note that the service sector selling prices of Goods G1 and G2 to the domestic final demand sector, $p_{G1}{}^{SF}$ and $p_{G2}{}^{SF}$, are somewhat higher than the corresponding import purchase prices for these good, $p_{G1m}{}^{SR}$ and $p_{G2}{}^{SF}$, but this is natural: the service sector must make a positive margin on its trading in these commodities in order to cover the costs of storage and distribution.

20.104 The Service Industry obviously contains elements of the traditional storage, wholesaling and retailing industries. The treatment of these industries that is followed in the artificial data example is a *gross output treatment* as opposed to a *margin industry treatment*. In the gross output treatment, goods for resale are purchased and the full purchase price times the amount purchased appears as an intermediate input cost and then the goods are sold subsequently at a higher price and this selling price times the amount sold appears as a contribution to gross output. In the margin treatment, it is assumed that the amount sold during the accounting period is at least roughly equal to the amount purchased, and the difference between the selling price and the purchase price (the margin) is multiplied by the amount bought and sold and is treated as a gross output with no corresponding intermediate input cost. Thus for the case of an imported good, if the margin treatment of wholesaling/retailing/storage (WRS) output is used, the margin would be credited to this WRS industry and the full import price plus the margin would appear as an intermediate input by the purchasing industry (or final demand sector). Thus the margin treatment of the WRS industry would be similar to the margin treatment that has been accorded to the transportation industry. However, there is a difference between the WRS industry and the transportation industry: for the transportation industry, one can be fairly certain that the goods "purchased" by the transport industry are equal to the goods "sold" by the industry and the margin treatment is perfectly justified. This is not necessarily the case for the WRS industry: sales are not necessarily equal to purchases in any given accounting period. Thus it seems preferable to use the gross output treatment for these distributive industries over the margin approach, although individual countries may feel that sales are sufficiently close to purchases so that the margin approach is a reasonable approximation to the actual situation and hence can be used in their national accounts.[43]

20.105 The fifteen commodity tax series that form part of the Industry S taxes listed in Tables 20.16-20.19 are listed for 5 periods in Table 20.24.

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[43] For theoretical treatments of the accounting problems associated with measuring the contribution of inventories to retailing and wholesaling production, see paragraphs 6.57-6.79 of *SNA 1993*, Diewert and Smith (1994), Ehemann (2005), Diewert (2005) and Hill (2005).

Table 20.24 Industry S Commodity Taxes

Commodity	G1 t_{G1}^{SF}	G2 t_{G2}^{SF}	S1 t_{S1}^{SG}	S1 t_{S1}^{ST}	S1 t_{S1}^{SF}	S2 t_{S2}^{SG}	S2 t_{S2}^{ST}	S2 t_{S2}^{SF}
1	0.02	0.17	0.01	0.01	0.10	0.15	0.15	0.30
2	0.05	0.23	0.02	0.02	0.15	0.11	0.11	0.25
3	0.02	0.19	0.03	0.03	0.18	0.08	0.08	0.20
4	0.06	0.17	0.03	0.03	0.19	0.05	0.05	0.10
5	0.07	0.24	0.03	0.02	0.20	0.02	0.02	0.05

G4 t_{G4}^{GS}	T t_{T}^{TS}	S1 t_{S1x}^{SR}	G1 t_{G1m}^{SR}	G2 t_{G2m}^{SR}	S1 t_{S1m}^{SR}	T t_{Tm}^{SR}
0	0	0	0.02	0.15	0.05	0.03
0	0	0	0.03	0.20	0.06	0.04
0	0	0	0.04	0.25	0.09	0.04
0	0	0	0.04	0.20	0.09	0.03
0	0	0	0.04	0.25	0.10	0.03

20.106 Note that the tax rates on domestic intermediate inputs used by Industry S are all set equal to zero under the convention used in this chapter that the selling industry pays any applicable commodity taxes.[44]

20.107 The fifteen quantity series that form part of the Industry S data in Tables 20.16-20.19 are listed for 5 periods in Table 20.25.

Table 20.25 Industry S Quantities of Outputs and Inputs

	G1 y_{G1}^{SF}	G2 y_{G2}^{SF}	S1 y_{S1}^{SG}	S1 y_{S1}^{ST}	S1 y_{S1}^{SF}	S2 y_{S2}^{SG}	S2 y_{S2}^{ST}	S2 y_{S2}^{SF}
1	10	8	4	2.0	15	2	1.1	3.0
2	11	9	5	2.5	20	4	1.5	4.3
3	12	9	6	3.0	25	8	2.1	6.5
4	13	10	8	3.5	33	14	3.5	10.5
5	14	11	10	3.5	40	20	5.0	15.0

G4 y_{G4}^{GS}	T y_{T}^{TS}	S1 x_{S1x}^{SR}	G1 m_{G1m}^{SR}	G2 m_{G2m}^{SR}	S1 m_{S1m}^{SR}	T m_{Tm}^{SR}
5	1.0	14	10	10	3	1.0
6	1.1	19	11	11	4	1.5
7	1.2	24	12	11	6	1.7
7	1.3	31	13	12	9	1.9

[44] The selling industry also receives any applicable commodity subsidies.

8	1.3	42	14	13	13	2.0

The quantities of Industry S deliveries to Industry G, y_{S1}^{SG} and y_{S2}^{SG}, and the quantities of deliveries of good G4 from Industry G to Industry S, y_{G4}^{GS}, are exactly the same in Tables 20.22 and 20.25.

20.108 Note that y_{G1}^{SF}, the quantity of imported food G1 sold by Industry S to domestic final demanders F, is exactly equal to m_{G1m}^{SR}, imports of food into Industry S. However, y_{G2}^{SF}, the quantity of imported energy products G2 sold by Industry S to domestic final demanders, is less than the quantity of energy imported by Industry S, m_{G2m}^{SR}. The reason for this difference is that Industry S uses some of the imported energy for heat and other purposes as it supplies services to other sectors of the economy. Sales by Industry S of traditional services S1 to Industry G, Industry T, domestic final demand F and to the rest of the world R, y_{S1}^{SG}, y_{S1}^{ST}, y_{S1}^{SF} and x_{S1x}^{SR} respectively, all increase quite rapidly, doubling or tripling over the 5 periods. Imports of traditional services S1 into Industry S, m_{S1m}^{SR}, increase even more rapidly, growing from 3 to 13 over the 5 periods. The sales of high tech services by Industry S to Industries G and T and to the domestic final demand sector F, y_{S2}^{SG}, y_{S2}^{ST} and y_{S2}^{SF} respectively, all increase very rapidly, growing between 5 and 10 fold over the 5 periods. The quantities of domestic intermediate inputs of good G4, y_{G4}^{GS}, and of the transportation service, y_T^{TS}, used by Industry S grew fairly slowly over the 5 periods. Imported transportation services, m_{Tm}^{SR}, associated with the importation of G1 and G2 by Industry S, doubled over the 5 periods.

B.6 Industry T Prices and Quantities

20.109 The nine final demand price series that form part of the Industry T data in Tables 20.16-20.19 are listed for 5 periods in Table 20.26.

Table 20.26 Industry T Final Demand Prices

Period	T	T	T	G4	S1	S2	T	G2	T
	p_T^{TG}	p_T^{TS}	p_T^{TF}	p_{G4}^{GT}	p_{S1}^{ST}	p_{S2}^{ST}	p_{Tx}^{TR}	p_{G2m}^{TR}	p_{Tm}^{TR}
1	1.0	1.0	1.2	0.9	1.0	1.0	1.1	1.0	1.0
2	1.6	1.6	1.8	1.2	1.4	0.8	1.7	2.1	1.6
3	1.5	1.5	1.7	1.5	1.7	0.6	1.5	1.6	1.4
4	1.3	1.3	1.6	1.55	1.8	0.4	1.3	1.1	1.2
5	1.8	1.8	2.2	1.6	1.9	0.2	1.8	2.2	1.8

20.110 The entries for p_T^{TS}, p_{S1}^{ST} and p_{S2}^{ST} in Tables 20.26 and 20.23 are the same series as are the entries for p_T^{TG} and p_{G4}^{GT} in Tables 20.26 and 20.20. Again, this reflects the fact that the sellers and purchasers of domestic intermediate inputs pay and receive the same amounts of money for their cross industry purchases and sales.

20.111 The industry selling prices for transportation services shows much the same trends across all destinations. The selling prices of transportation services to domestic final demand, p_T^{TF}, are higher than the other selling prices due to higher commodity taxation for deliveries to final demand.

20.112 The commodity tax series that form part of the Industry T taxes listed in Tables 20.16-20.19 are listed for 5 periods in Table 20.27.

Table 20.27 Industry T Commodity Taxes

	T	T	T	G4	S1	S2	T	G	T
Period	t_T^{TG}	t_T^{TS}	t_T^{TF}	t_{G4}^{GT}	t_{S1}^{ST}	t_{S2}^{ST}	t_{Tx}^{TR}	t_{G2m}^{TR}	t_{Tm}^{TR}
1	0.01	0.01	0.10	0	0	0	0	0.15	0.03
2	0.02	0.02	0.15	0	0	0	0	0.20	0.04
3	0.03	0.03	0.18	0	0	0	0	0.25	0.04
4	0.03	0.03	0.19	0	0	0	0	0.20	0.03
5	0.03	0.03	0.20	0	0	0	0	0.25	0.03

20.113 The commodity taxes on deliveries of transportation services to Industries G and S, t_T^{TG} and t_T^{TS} respectively, are small but the taxes on deliveries to the final demand sector, t_T^{TF}, are fairly substantial, as are the taxes on the transportation sector's imports of energy products, t_{G2m}^{TR}. By convention, any taxes on Industry T's use of domestic intermediate inputs are paid by the selling industry so t_{G4}^{GT}, t_{S1}^{ST} and t_{S2}^{ST} are all 0. There are no taxes on the export of transportation services in this economy so that t_{Tx}^{TR} is 0 as well. There are small taxes on Industry T's importation of transport services, t_{Tm}^{TR}.

20.114 The nine final demand quantity series that form part of the Industry T data in Tables 20.16-20.19 are listed for 5 periods in Table 20.28.

Table 20.28 Industry T Quantities of Outputs and Inputs

	T	T	T	G4	S1	S2	T	G2	T
Period	y_T^{TG}	y_T^{TS}	y_T^{TF}	y_{G4}^{GT}	y_{S1}^{ST}	y_{S2}^{ST}	x_{Tx}^{TR}	m_{G2m}^{TR}	m_{Tm}^{TR}
1	3	1.0	5	2	2.0	1.1	3	3	1.5
2	3.5	1.1	5	2.5	2.5	1.5	4	3	1.7
3	4	1.2	6	3	3.0	2.1	5	3.5	2.2
4	5	1.3	7	3.5	3.5	3.5	5.5	4	2.4
5	6	1.3	7	4	3.5	5.0	6	4.5	2.5

20.115 The entries for y_T^{TS}, y_{S1}^{ST} and y_{S2}^{ST} in Tables 20.28 and 20.25 are the same series as are the entries for y_T^{TG} and y_{G4}^{GT} in Tables 20.28 and 20.22. All transportation service inputs and outputs grow relatively smoothly and roughly double over the 5 periods.

20.116 This completes the listing of the basic price, tax and quantity data that will be used in subsequent sections of this chapter in order to illustrate how various index number formulae differ and how consistent sets of producer price indices can be formed in a set of production accounts that are roughly equivalent to the production accounts that are described in Chapter 15 of *SNA 1993*.

D. The Artificial Data Set for Domestic Final Demand

D.1 The Final Demand Data Set

20.117 In order to illustrate what kind of differences can result from the choice of different index number formulae, the price and quantity data that correspond to domestic deliveries to final demand that were listed in the previous section are used as a test data set in this section. The 6 final demand price series are listed in Table 20.29 and the corresponding quantity series are listed in Table 20.30.

Table 20.29 Prices for Six Domestic Final Demand Commodities

	G1	G2	G4	S1	S2	T
	Food	Energy	Goods	Services	High Tech Ser	Transport
Period t	p_1^t	p_2^t	p_3^t	p_4^t	p_5^t	p_6^t
1	1.2	1.4	1.0	1.3	1.15	1.2
2	1.5	2.8	1.3	1.8	0.94	1.8
3	1.2	2.2	1.6	2.2	0.72	1.7
4	1.6	1.5	1.65	2.4	0.45	1.6
5	1.7	3.0	1.7	2.6	0.23	2.2

20.118 The prices p_1^t, p_2^t, p_3^t, p_4^t, p_5^t and p_6^t in Table 20.30 correspond to the final demand prices p_{G1}^{SF}, p_{G2}^{SF}, p_{G4}^{GF}, p_{S1}^{SF}, p_{S2}^{SF} and p_T^{TF} respectively, which are listed in Tables 20.20, 20.23 and 20.26.

Table 20.30 Quantities for Six Domestic Final Demand Commodities

	G1	G2	G4	S1	S2	T
	Food	Energy	Goods	Services	High Tech Ser	Transport
Period t	q_1^t	q_2^t	q_3^t	q_4^t	q_5^t	q_6^t
1	10	8	35	15	3.0	5
2	11	9	40	20	4.3	5
3	12	9	45	25	6.5	6
4	13	10	49	33	10.5	7
5	14	11	55	40	15.0	7

20.119 The quantities q_1^t, q_2^t, q_3^t, q_4^t, q_5^t and q_6^t in Table 20.30 correspond to the final demand quantities y_{G1}^{SF}, y_{G2}^{SF}, y_{G4}^{GF}, y_{S1}^{SF}, y_{S2}^{SF} and y_T^{TF} respectively, which are listed in Tables 20.22, 20.25 and 20.28.

20.120 It is useful to also list the period t expenditures on all six domestic finally demanded commodities, $p^t \cdot q^t$, along with the corresponding expenditure shares, s_1^t, ... , s_6^t; see Table 20.31.

Table 20.31 Total Expenditures and Expenditure Shares for Six Domestic Final Demand Commodities

Period t	Expenditures $p^t \cdot q^t$	Food s_1^t	Energy s_2^t	Goods s_3^t	Services s_4^t	H.T. Services s_5^t	Transport Services s_6^t
1	87.150	0.1377	0.1285	0.4016	0.2238	0.0396	0.0688
2	142.742	0.1156	0.1765	0.3643	0.2522	0.0283	0.0631
3	176.080	0.0818	0.1124	0.4089	0.3124	0.0266	0.0579
4	211.775	0.0982	0.0708	0.3818	0.3740	0.0223	0.0529
5	273.150	0.0871	0.1208	0.3423	0.3807	0.0126	0.0564

20.121 The expenditure shares for food, goods and high tech services decrease markedly over the 5 periods, the share for transport services decreases somewhat, the share of energy stays roughly constant but with substantial period to period fluctuations and the share of general services increases substantially.

20.122 Note that the price of food and energy fluctuates considerably from period to period but the quantities demanded tend to trend upwards at a fairly smooth rate, reflecting the low elasticity of price demand for these products. The fluctuations in energy prices tends to produce similar fluctuations in the price of domestic transportation services but the fluctuations in price are more damped in the case of transport services. The price of goods tends up fairly rapidly in periods 2 and 3 but then the rate of increase falls off. The corresponding quantity trends upwards fairly steadily. The price of traditional services, p_4^t, increases rapidly in periods 2 and 3 and then increases more slowly. Overall, the price of traditional services increases more rapidly than the price of goods but the quantity of services demanded q_4^t increases more rapidly than the quantity of goods, q_3^t. The price of high technology services, p_5^t, decreases rapidly over the 5 periods, falling to about 1/5 of the initial price level. The corresponding quantities demanded, q_5^t, increase rapidly, increasing five fold over the sample period. Thus overall, the data set exhibits a rather wide variety of trends in prices and quantities but these trends are not unrealistic in today's world economy. The movements of prices and quantities in this artificial data set are more pronounced than the year to year movements that would be encountered in a typical country but they do illustrate the problem that is facing compilers of producer and consumer price indices: namely, *year to year price and quantity movements are far from being proportional across commodities so the choice of index number formula will matter*.

D.2 Some Familiar Index Number Formulae

20.123 Every price statistician is familiar with the *Laspeyres index*, P_L, and the *Paasche index*, P_P, defined in Chapter 15 above. These indices are listed in Table 20.32 along with the two main unweighted indices that were considered in Chapters 10, 17 and 21: the *Carli index* and the *Jevons index*. The indices in Table 20.31 compare the prices in period *t* with the prices in period 1; i.e., they are *fixed base indices*. Thus the period *t* entry for the Carli index, P_C, is simply the arithmetic mean of the 8 price relatives, $\sum_{i=1}^{6} (1/6)(p_i^t/p_i^1)$, while the period

t entry for the Jevons index, P_J, is the geometric mean of the 6 long term price relatives, $\prod_{i=1}^{6} (p_i^t/p_i^1)^{1/6}$.

Table 20.32 The Fixed Base Laspeyres, Paasche, Carli and Jevons Indices

Period t	P_L^t	P_P^t	P_C^t	P_J^t
1	1.0000	1.0000	1.0000	1.0000
2	1.3967	1.3893	1.3753	1.3293
3	1.4832	1.4775	1.3177	1.2478
4	1.5043	1.4916	1.2709	1.1464
5	1.7348	1.6570	1.5488	1.2483

20.124 Note that by period 5, the spread between the fixed base Laspeyres and Paasche price indices is not negligible: P_L is equal to 1.7348 while P_P is 1.6570*, a spread of about 4.7 percent*. Since both of these indices have exactly the same *theoretical* justification, it can be seen that the choice of index number formula matters. There is also a substantial spread between the two unweighted indices by period 5: the fixed base Carli index is equal to 1.5488, while the fixed base Jevons index is 1.2483, *a spread of about 24 percent*. However, more troublesome than this spread is the fact that *the unweighted indices are well below both the Paasche and Laspeyres indices* by period 5. Thus when there are divergent trends in both prices and quantities, it will usually be the case that unweighted price indices will give very different answers than their weighted counterparts. Since none of the index number theories considered in previous chapters supported the use of unweighted indices, their use is not recommended for aggregation at the "higher level," that is, when data on weights are available. However, in Chapter 21 aggregation at the "lower level" is considered for which weights are unavailable and the use of unweighted index number formulas will be revisited.. Finally, note that the Jevons index is always considerably below the corresponding Carli index. This will always be the case (unless prices are proportional in the two periods under consideration) because a geometric mean is always equal to or less than the corresponding arithmetic mean.[45]

20.125 It is of interest to recalculate the four indices listed in Table 20.32 above using *the chain principle* rather than the *fixed base principle*. Our expectation is that the spread between the Paasche and Laspeyres indices will be reduced by using the chain principle. These chained indices are listed in Table 20.33.

Table 20.33 The Chained Laspeyres, Paasche, Carli and Jevons Indices

Period t	P_L^t	P_P^t	P_C^t	P_J^t
1	1.0000	1.0000	1.0000	1.0000
2	1.3967	1.3893	1.3753	1.3293
3	1.4931	1.4952	1.3178	1.2478

[45]This is the Theorem of the Arithmetic and Geometric Mean; see Hardy, Littlewood and Polyá (1934) and Chapter 20.

4	1.5219	1.5219	1.2527	1.1464
5	1.7176	1.7065	1.4745	1.2483

20.126 It can be seen comparing Tables 20.32 and 20.33 that chaining eliminated most of the spread between the fixed base Paasche and Laspeyres indices for period 5; i.e., the spread between the chained Laspeyres and Paasche indices has dropped from 4.7% to 0.6%. Note that chaining did not affect the Jevons index. This is an advantage of the index but the lack of weighting is a fatal flaw. The "truth" would be expected to lie between the Paasche and Laspeyres indices but from Tables 20.32 and 20.33, the unweighted Jevons index is far below this acceptable range. The fixed base and chained Carli indices also lie outside this acceptable range.

D.3 Asymmetrically Weighted Index Number Formulae

20.127 A systematic comparison of all of *the asymmetrically weighted price indices* is now undertaken. The *fixed base indices* are listed in Table 20.34. The fixed base *Laspeyres* and *Paasche indices*, P_L and P_P, are the same as those indices listed in Table 20.32 above. The *Palgrave index*, P_{PAL}, is defined by equation (16.55). The indices denoted by P_{GL} and P_{GP} are *the geometric Laspeyres and geometric Paasche indices*[46] which were defined Chapter 16. For *the geometric Laspeyres index*, P_{GL}, the weights for the price relatives are the *base period expenditure shares*, s_i^1. This index should be considered an alternative to the fixed base Laspeyres index since each of these indices makes use of the same information set. For *the geometric Paasche index*, P_{GP}, the weights for the price relatives are the *current period expenditure shares*, s_i^t. Finally, the index P_{HL} *is the harmonic Laspeyres index* that was defined by (16.59).

Table 20.34 Asymmetrically Weighted Fixed Base Indices

Period t	P_{PAL}^t	P_{GP}^t	P_L^t	P_{GL}^t	P_P^t	P_{HL}^t
1	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
2	1.4381	1.4129	1.3967	1.3743	1.3893	1.3527
3	1.5400	1.5145	1.4832	1.4477	1.4775	1.3995
4	1.6064	1.5650	1.5043	1.4469	1.4916	1.3502
5	1.8316	1.7893	1.7348	1.6358	1.6570	1.3499

20.128 By looking at the period 5 entries in Table 20.34, it can be seen that the spread between all of these fixed base asymmetrically weighted indices has increased to be much larger than our earlier spread of 4.7 percent between the fixed base Paasche and Laspeyres indices. In Table 20.34, the period 5 Palgrave index is about 1.36 times as big as the period 5 harmonic Laspeyres index, P_{HL}. Again, *this illustrates the point that due to the non-*

[46] Vartia (1978, p. 272) used the terms *logarithmic Laspeyres* and *logarithmic Paasche*, respectively.

proportional growth of prices and quantities in most economies today, the choice of index number formula is very important.[47]

20.129 It is possible to explain why certain of the indices in Table 20.34 are bigger than others. When all weights are positive, it can be shown that a *weighted arithmetic mean* of *N* numbers is equal to or greater than the corresponding *weighted geometric mean* of the same *N* numbers which in turn is equal to or greater than the corresponding *weighted harmonic mean* of the same N numbers.[48] It can be seen that the three indices P_{PAL}, P_{GP}, and P_P all use the current period expenditure shares s_i^t to weight the price relatives (p_i^t/p_i^1) but P_{PAL} is a weighted *arithmetic* mean of these price relatives, P_{GP} is a weighted *geometri*c mean of these price relatives and P_P is a weighted *harmonic* mean of these price relatives. Thus since there are no negative components in final demand, by Schlömilch's inequality,:[49]

(20.31) $P_{PAL} \geq P_{GP} \geq P_P$.

20.130 Viewing Table 20.34, it can be seen that the inequalities (20.31) hold for all periods. It can also be verified that the three indices P_L, $P_{GL,}$ and P_{HL} all use the base period expenditure shares s_i^1 to weight the price relatives (p_i^t/p_i^1) but P_L is a weighted *arithmetic* mean of these price relatives, P_{GL} is a weighted *geometri*c mean of these price relatives, and P_{HL} is a weighted *harmonic* mean of these price relatives. Since all of the expenditure shares are positive, then by Schlömilch's inequality,:[50]

(20.32) $P_L \geq P_{GL} \geq P_{HL}$.

Viewing Table 20.34, it can be seen that the inequalities (20.32) hold for all periods.

20.131 Now continue with the systematic comparison of all of *the asymmetrically weighted price indices*. These indices using the *chain principle* are listed in Table 20.35.

Table 20.35 Asymmetrically Weighted Chained Indices

Period t	P_{PAL}^t	P_{GP}^t	P_L^t	P_{GL}^t	P_P^t	P_{HL}^t
1	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
2	1.4381	1.4129	1.3967	1.3743	1.3893	1.3527
3	1.6019	1.5488	1.4931	1.4400	1.4952	1.3870
4	1.6734	1.5987	1.5219	1.4461	1.5219	1.3690

---

[47] Allen and Diewert (1981) showed that the Paasche, Laspeyres and Fisher indices will all be equal if either prices or quantities move in a proportional manner over time. Thus in order to get a spread between the Paasche and Laspeyres indices, it is required that *both* prices and quantities move in a nonproportional manner.

[48]This follows from Schlömilch's (1858) inequality; see Hardy, Littlewood and Polyá (1934, chapter 11).

[49]These inequalities were noted by Fisher (1922, p. 92) and Vartia (1978, p. 278).

[50]These inequalities were also noted by Fisher (1922, p. 92) and Vartia (1978, p. 278).

5	1.9802	1.8375	1.7176	1.5954	1.7065	1.4788

20.132 Viewing Table 20.35, it can be seen that the use of the chain principle only marginally reduced the spread between all of the asymmetrically weighted indices compared to their fixed base counterparts in Table 20.34. For period 5, the spread between the smallest and largest asymmetrically weighted fixed base index was 35.7 percent but for the period 5 chained indices, this spread was marginally reduced to 33.9 percent.

D.4 Symmetrically Weighted Index Number Formulae

20.133 Symmetrically weighted indices can be decomposed into two classes: *superlative indices* and *other symmetrically weighted indices*. Superlative indices have a close connection to economic theory; i.e., as was seen in Chapter 18, a superlative index is exact for a representation of the producer's production function or the corresponding unit revenue function that can provide a second order approximation to arbitrary technologies that satisfy certain regularity conditions. In Chapter 18 four primary superlative indices were considered:

- the *Fisher ideal price index, P_F*, defined by (18.9);
- the *Walsh price index, P_W*, defined by (16.19) (this price index also corresponds to the quantity index Q^1 defined by (18.26) in Chapter 18);
- the *Törnqvist-Theil price index, P_T*, defined by (18.10) and
- the *implicit Walsh price index, P_{IW}*, that corresponds to the Walsh quantity index Q_W defined by (17.34) (this is also the index P^1 defined by (18.31)).

20.134 These four symmetrically weighted superlative price indices are listed in Table 20.19 using the fixed base principle. Also listed in this table are two symmetrically weighted price indices:[51]

- the Marshall Edgeworth price index, P_{ME}, defined by (15.18) and
- the Drobisch price index, P_D, the arithmetic average of the Paasche and Laspeyres price indices.

Table 20.36 Symmetrically Weighted Fixed Base Indices

Period t	P_T^t	P_{IW}^t	P_W^t	P_F^t	P_D^t	P_{ME}^t
1	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000
2	1.39347	1.39312	1.39307	1.39297	1.39298	1.39267
3	1.48073	1.48219	1.48129	1.48034	1.48034	1.47990
4	1.50481	1.50627	1.50216	1.49796	1.49797	1.49645
5	1.71081	1.72041	1.70612	1.69545	1.69589	1.68389

[51] Diewert (1978; 897) showed that the Drobisch Sidgwick Bowley price index approximates any superlative index to the second order around an equal price and quantity point; i.e., P_{SB} is a *pseudo-superlative index*. Straightforward computations show that the Marshall Edgeworth index P_{ME} is also pseudo-superlative.

20.135 Note that the Drobisch index P_D is always equal to or greater than the corresponding Fisher index P_F. This follows from the facts that the Fisher index is the geometric mean of the Paasche and Laspeyres indices while the Drobisch index is the arithmetic mean of the Paasche and Laspeyres indices and an arithmetic mean is always equal to or greater than the corresponding geometric mean. Comparing the fixed base asymmetrically weighted indices, Table 20.34, with the symmetrically weighted indices, Table 20.36, *it can be seen that the spread between the lowest and highest index in period 5 is much less for the symmetrically weighted indices*. The spread was 1.8316/1.3499 = 1.357 for the asymmetrically weighted indices but only 1.72041/1.68389 = 1.022 for the symmetrically weighted indices. If the analysis is restricted to the superlative indices listed for period 5 in Table 20.19, then this spread is further reduced to 1.72041/1.69545 = 1.015; i.e., the spread between the fixed base superlative indices is only 1.5 percent compared to the fixed base spread between the Palgrave and Harmonic Laspeyres indices of 35.7 percent (1.8316/1.3499 = 1.357). The spread between the superlative indices can be expected to be further reduced by using the chain principle.

20.136 The symmetrically weighted indices are recomputed using the chain principle. The results may be found in Table 20.37. A quick glance at Table 20.20 shows that *the combined effect of using both the chain principle as well as symmetrically weighted indices is to dramatically reduce the spread between all indices constructed using these two principles*. The spread between all of the symmetrically weighted indices in period 5 is only 1.7127/1.7116 = 1.0006 or 0.06 percent, which is negligible.

Table 20.37 Symmetrically Weighted Chained Indices

Period t	P_T^t	P_{IW}^t	P_W^t	P_F^t	P_D^t	P_{ME}^t
1	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
2	1.3935	1.3931	1.3931	1.3930	1.3930	1.3927
3	1.4934	1.4941	1.4945	1.4941	1.4941	1.4942
4	1.5205	1.5219	1.5224	1.5219	1.5219	1.5218
5	1.7122	1.7122	1.7127	1.7120	1.7121	1.7116

20.137 The results listed in Table 20.37 reinforce the numerical results tabled in Hill (2006) and Diewert (1978, p. 894): *the most commonly used chained superlative indices will generally give approximately the same numerical results*.[52] This numerical approximation property holds in spite of the erratic nature of the fluctuations in the data in Tables 20.29-20.31. In particular, the chained Fisher, Törnqvist and Walsh indices will generally approximate each other very closely.

[52]More precisely, the superlative quadratic mean of order r price indices P^r defined by (17.84) and the implicit quadratic mean of order r price indices P^{r*} defined by (17.81) will generally closely approximate each other provided that r is in the interval $0 \leq r \leq 2$.

D.5 Superlative Indices and Two Stage Aggregation

20.138 Attention is now turned to the differences between superlative indices and their counterparts that are constructed in two stages of aggregation; see section D.6 of Chapter 18 for a discussion of the issues and a listing of the formulas used. In the artificial data set for domestic final demand, the first three commodities are aggregated into a *goods aggregate* and the final three commodities are aggregated into a *services aggregate*. In the second stage of aggregation, the good and services components will be aggregated into a domestic final demand price index.

20.139 The results of single stage and two stage aggregation are reported in Table 20.38 using period 1 as the *fixed base* for the Fisher index P_F, the Törnqvist index P_T and the Walsh and implicit Walsh indexes, P_W and P_{IW}.

Table 20.38 Single Stage and Two Stage Fixed Base Superlative Indices

Period t	P_F^t	P_{F2S}^t	P_T^t	P_{T2S}^t	P_W^t	P_{W2S}^t	P_{IW}^t	P_{IW2S}^t
1	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
2	1.3930	1.3931	1.3935	1.3935	1.3931	1.3931	1.3931	1.3932
3	1.4803	1.4808	1.4807	1.4800	1.4813	1.4813	1.4822	1.4821
4	1.4980	1.4998	1.5048	1.5003	1.5022	1.5021	1.5063	1.5051
5	1.6954	1.7012	1.7108	1.7007	1.7061	1.7063	1.7204	1.7176

20.140 Viewing Table 20.237, it can be seen that the fixed base single stage superlative indices generally approximate their fixed base two stage counterparts fairly closely. The divergence between the single stage Törnqvist index P_T and its two stage counterpart P_{T2S} in period 5 is 1.7108/1.7007 = 1.006 or 0.6 percent. The other divergences are even less.

20.141 Using *chained indices*, the results are reported in Table 20.39 for the two stage aggregation procedure. Again, the single stage and their two stage counterparts are listed for the Fisher index P_F, the Törnqvist index P_T and the Walsh and implicit Walsh indexes, P_W and P_{IW}.

Table 20.39 Single Stage and Two Stage Chained Superlative Indices

Period t	P_F^t	P_{F2S}^t	P_T^t	P_{T2S}^t	P_W^t	P_{W2S}^t	P_{IW}^t	P_{IW2S}^t
1	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
2	1.3930	1.3931	1.3935	1.3935	1.3931	1.3931	1.3931	1.3932
3	1.4941	1.4943	1.4934	1.4942	1.4945	1.4944	1.4941	1.4945
4	1.5219	1.5221	1.5205	1.5218	1.5224	1.5223	1.5219	1.5226
5	1.7120	1.7125	1.7122	1.7136	1.7127	1.7127	1.7122	1.7132

20.35

20.174 Viewing Table 20.39, it can be seen that the chained single stage superlative indices generally approximate their fixed base two stage counterparts quite closely. The divergence between the chained Törnqvist index P_T and its two stage counterpart P_{T2S} in period 5 is 1.7136/1.7122 = 1.0008 or 0.08 percent. The other divergences are all less than this. Given the large dispersion in period to period price movements, these two stage aggregation errors are not large. However, the important point that emerges from Table 20.39 is that *the use of the chain principle has reduced the spread between all 8 single stage and two stage superlative indices* compared to their fixed base counterparts in Table 20.38. The maximum spread for the period 5 chained index values is 0.09 percent while the maximum spread for the period 5 fixed base index values is 1.5 percent.

20.175 The final formulas that is illustrated using the artificial final expenditures data set are the *additive percentage change decompositions* for the Fisher ideal index that were discussed in section B.8 of Chapter 16. The *chain links* for the Fisher price index will first be decomposed using the Diewert (2002) decomposition formulas (16.41) to (16.43). The results of the decomposition are listed in Table 20.40. Thus $P_F - 1$ is *the percentage change in the Fisher ideal chain link* going from period $t-1$ to t and *the decomposition factor* $v_{Fi}\Delta p_i = v_{Fi}(p_i^t - p_i^{t-1})$ is the contribution to the total percentage change of the change in the ith price from p_i^{t-1} to p_i^t for $i = 1,2,\ldots,6$.

Table 20.40 The Diewert Additive Percentage Change Decomposition of the Fisher Index

Period t	$P_F^t - 1$	$v_{F1}^t\Delta p_1^t$	$v_{F2}^t\Delta p_2^t$	$v_{F3}^t\Delta p_3^t$	$v_{F4}^t\Delta p_4^t$	$v_{F5}^t\Delta p_5^t$	$v_{F6}^t\Delta p_6^t$
2	0.3930	0.0331	0.1253	0.1185	0.0928	-0.0082	0.0314
3	0.0726	-0.0225	-0.0353	0.0831	0.0586	-0.0077	-0.0036
4	0.0186	0.0261	-0.0347	0.0123	0.0301	-0.0118	-0.0034
5	0.1250	0.0059	0.0693	0.0114	0.0321	-0.0123	0.0185

20.176 Viewing Table 20.40, it can be seen that the price index going from period 1 to 2 grew 39.30 percent and the contributors to this change were the increases in the price of commodity 1, finally demanded agricultural products (3.31 percentage points); commodity 2, finally demanded energy (12.53 percentage points); commodity 3, finally demanded goods (11.85 percentage points); commodity 4, traditional services (9.28 percentage points) and commodity 6, transportation services (3.14 percentage points). High technology services, commodity 5, decreased in price and this fall in prices subtracted 0.82 percentage points from the overall Fisher price index going from period 1 to 2. The sum of the last six entries for period 2 in Table 20.40 is equal to .3930, the percentage increase in the Fisher price index going from period 1 to 2. It can be seen that a big price change in a particular component i combined with a big expenditure share in the two periods under consideration will lead to a big decomposition factor, $v_{Fi}\Delta p_i$.

20.177 Our final set of computations illustrate the *additive percentage change decomposition* for the Fisher ideal index that is due to Van IJzeren (1987, p. 6) that was mentioned in section C.8 of Chapter 16.[53] First, the Fisher price index going from period t−1 to t is written in the following form:

[53]It was also independently derived by Dikhanov (1997) and used by Ehemann, Katz, and Moulton (2002).

(20.33) $P_F(p^{t-1},p^t,q^{t-1},q^t) = \dfrac{\sum_{i=1}^{6} q_{Fi}^{*} p_i^t}{\sum_{i=1}^{6} q_{Fi}^{*} p_i^{t-1}}$

where the reference quantities need to be defined somehow. Van IJzeren (1987; 6) showed that the following reference weights provided an *exact additive representation for the Fisher ideal price index*:

(20.34) $q_{Fi}{}^{*} \equiv (1/2)q_i^{t-1} + [(1/2)Q_F(p^{t-1},p^t,q^{t-1},q^t)]$; $\qquad i = 1,2,...,6$

where Q_F is the overall Fisher quantity index. Thus using the Van IJzeren quantity weights $q_{Fi}{}^{*}$, the *following Van IJzeren additive percentage change decomposition for the Fisher price index* is obtained:

(20.35) $P_F(p^0,p^1,q^0,q^1) - 1 = \dfrac{\sum_{i=1}^{6} q_{Fi}^{*} p_i^t}{\sum_{i=1}^{6} q_{Fi}^{*} p_i^{t-1}} - 1$

$$= \sum_{i=1}^{6} v_{Fi}^{t*}(p_i^t - p_i^{t-1})$$

where the *Van IJzeren weight* for commodity i, $v_{Fi}{}^{t*}$, is defined as

(20.36) $v_{Fi}{}^{t*} \equiv \dfrac{\sum_{i=1}^{6} q_{Fi}^{*}}{\sum_{i=1}^{6} q_{Fi}^{*} p_i^{t-1}}$; $\qquad i = 1,2,...,6.$

20.178 The *chain links* for the Fisher price index will be decomposed using the formulas (20.33) to (20.36) listed above. The results of the decomposition are listed in Table 20.41. Thus $P_F - 1$ is *the percentage change in the Fisher ideal chain link* going from period $t - 1$ to t and *the Van IJzeren decomposition factor* $v_{Fi}{}^{t*}\Delta p_i{}^t$ is the contribution to the total percentage change of the change in the ith price from p_i^{t-1} to p_i^t for $i = 1,2,\ldots,6$.

Table 20.41 The Van IJzeren Additive Percentage Change Decomposition of the Fisher Index

Period t	$P_F^t - 1$	$v_{F1}^{t*}\Delta p_1^t$	$v_{F2}^{t*}\Delta p_2^t$	$v_{F3}^{t*}\Delta p_3^t$	$v_{F4}^{t*}\Delta p_4^t$	$v_{F5}^{t*}\Delta p_5^t$	$v_{F6}^{t*}\Delta p_6^t$
2	0.3930	0.0333	0.1256	0.1186	0.0917	-0.0080	0.0318
3	0.0726	-0.0226	-0.0354	0.0833	0.0586	-0.0077	-0.0036
4	0.0186	0.0261	-0.0347	0.0123	0.0301	-0.0118	-0.0034
5	0.1250	0.0059	0.0693	0.0114	0.0320	-0.0122	0.0185

20.179 Comparing the entries in Tables 20.40 and 20.41, it can be seen that the differences between the Diewert and Van IJzeren decompositions of the Fisher price index are *very small*.[54] This is somewhat surprising given the very different nature of the two decompositions.[55] As was mentioned in section C.8 of Chapter 16, the Van IJzeren decomposition of the chain Fisher *quantity* index is used by the Bureau of Economic Analysis in the U.S.[56]

E. National Producer Price Indices

E.1 The National Gross Domestic Output Price Index at Producer Prices

20.180 In this subsection and the following 3 subsections, national domestic gross output, export, domestic intermediate input and import price indices at producer prices (i.e., at basic prices for outputs and purchaser's prices for intermediate inputs) will be calculated using the data for each of the 3 industrial sectors listed in section B above. Only fixed base and chained Laspeyres, Paasche, Fisher and Törnqvist indices will be computed since these are the ones most likely to be used in practice.

20.181 It should be noted that the price indices computed in this section are appropriate ones to use for the calculation of business sector labour or multifactor productivity purposes.

20.182 The data listed in Tables 20.20-20.28 for Industries G, S and T are used to calculate fixed base Laspeyres, Paasche, Fisher and Törnqvist price indices for domestic outputs (at producer prices or basic prices in this case) for periods t equal 1 to 5, P_L^t, P_P^t, $P_F^{t,}$ and P_T^t, respectively. Producer prices are used in these computations (as opposed to final demand prices). There are 3 domestic output deliveries from Industry G, 8 domestic output deliveries from Industry S and 3 domestic output deliveries from Industry T so that each index is an aggregate of 14 separate series. The fixed base results are listed in Table 20.42.

Table 20.42 Fixed Base National Domestic Gross Output Price Indices at Producer Prices

Period t	P_L^t	P_P^t	P_F^t	P_T^t
1	1.0000	1.0000	1.0000	1.0000
2	1.3865	1.3735	1.3800	1.3810
3	1.4762	1.4459	1.4610	1.4650
4	1.4826	1.4203	1.4511	1.4683

[54] The maximum difference between the two tables occurs in period 2 for the p_4 contribution factor, which is 0.0928 in Table 20.40 and 0.0917 in Table 20.41.

[55] The terms in Diewert's decomposition can be given economic interpretations whereas the terms in the other decomposition are more difficult to interpret from the economic perspective. However, Reinsdorf, Diewert and Ehemann (2002) show that the terms in the two decompositions approximate each other to the second order around any point where the two price vectors are equal and where the two quantity vectors are equal.

[56] See Ehemann, Katz and Moulton (2002).

5	1.7017	1.5424	1.6201	1.6581

20.183 By period 5, the spread between the fixed base Laspeyres and Paasche national domestic output price indices is 1.7017/1.5424 = 1.103 or 10.3% and the spread between the Fisher and Törnqvist indices is 1.6581/1.6201 = 1.023 or 2.3%. In Table 20.43, the four indexes are recomputed using the chain principle. It is expected that the use of the chain principle will narrow the spreads between the various indices.

Table 20.43 Chained National Domestic Gross Output Price Indices at Producer Prices

Period t	P_L^t	P_P^t	P_F^t	P_T^t
1	1.0000	1.0000	1.0000	1.0000
2	1.3865	1.3735	1.3800	1.3810
3	1.4832	1.4728	1.4780	1.4783
4	1.4919	1.4759	1.4839	1.4839
5	1.6644	1.6328	1.6485	1.6500

20.184 An examination of the entries in Table 20.43 shows that chaining did indeed reduce the spread between the various index numbers. In period 5, the spread between the chained Laspeyres and Paasche national domestic output price indices is 1.6644/1.6328 = 1.019 or 1.9% and the spread between the chained Fisher and Törnqvist indices is 1.6500/1.6485 = 1.0009 or 0.09%, which is negligible considering the variation in the underlying data.

E.2 The National Export Price Index at Producer Prices

20.185 The data listed in Tables 20.20-20.28 for Industries G, S and T are used to calculate fixed base Laspeyres, Paasche, Fisher and Törnqvist price indices for all exported outputs (at producer prices or basic prices in this case), P_L^t, P_P^t, $P_F^{t,}$ and P_T^t, respectively. There is one exported good from each of the three industries so that each export price index is an aggregate of 3 separate series. The fixed base results are listed in Table 20.44.

Table 20.44 National Fixed Base Export Price Indices at Producer Prices

Period t	P_L^t	P_P^t	P_F^t	P_T^t
1	1.0000	1.0000	1.0000	1.0000
2	1.3181	1.3199	1.3190	1.3191
3	1.5826	1.5799	1.5812	1.5813
4	1.4766	1.4762	1.4764	1.4763
5	1.3672	1.3694	1.3683	1.3682

20.186 There is very little difference in any of the fixed base series listed in Table 20.44. The corresponding chained indices are listed below and are also very close to each other.

Table 20.45 National Chained Export Price Indices at Producer Prices

Period t	P_L^t	P_P^t	P_F^t	P_T^t
1	1.0000	1.0000	1.0000	1.0000
2	1.3181	1.3199	1.3190	1.3191

3	1.5786	1.5788	1.5787	1.5786
4	1.4717	1.4729	1.4723	1.4723
5	1.3690	1.3624	1.3657	1.3654

E.3 The National Domestic Intermediate Input Price Index at Producer Prices

20.187 The data listed in Tables 20.20-20.28 for Industries G, S and T are used to calculate fixed base Laspeyres, Paasche, Fisher and Törnqvist price indices for all domestic intermediate inputs (at producer prices or purchase prices in this case), P_L^t, P_P^t, P_F^t and P_T^t, respectively. There are 3 domestic intermediate inputs used in each of Industries G, S and T so that each domestic intermediate input price index is an aggregate of 8 separate series. The fixed base results are listed in Table 20.46.

Table 20.46 Fixed Base National Domestic Intermediate Input Price Indices at Producer Prices

Period t	P_L^t	P_P^t	P_F^t	P_T^t
1	1.0000	1.0000	1.0000	1.0000
2	1.3443	1.3053	1.3247	1.3265
3	1.4928	1.3441	1.4165	1.4324
4	1.4686	1.1836	1.3184	1.3619
5	1.5887	1.1306	1.3402	1.4268

20.188 The spread between the Laspeyres and Paasche fixed base indices is very large by period 5, equaling 1.5887/1.1306 = 1.405 or 40.5%. The spread between the Fisher and Törnqvist fixed base indices is not negligible either, equaling 1.4268/1.3402 = 1.065 or 6.5% in period 5. These relatively large spreads are due to the fact that the price of high tech services plummets over the sample period with corresponding large increases in quantities while the other prices increase substantially. As usual, we expect these spreads to diminish if the chained indices are used.

The corresponding chained indices are listed in Table 20.47.

Table 20.47 Chained National Domestic Intermediate Input Price Indices at Producer Prices

Period t	P_L^t	P_P^t	P_F^t	P_T^t
1	1.0000	1.0000	1.0000	1.0000
2	1.3443	1.3053	1.3247	1.3265
3	1.4765	1.4045	1.4400	1.4435
4	1.4217	1.3272	1.3736	1.3782
5	1.4573	1.3398	1.3973	1.4015

20.189 Chaining reduces the period 5 spread between Laspeyres and Paasche to 1.4573/1.3398 = 1.088 or 8.8% and between the Fisher and Törnqvist to 1.4015/1.3973 = 1.003 or 0.3%, which is an acceptable degree of divergence considering the volatility of the underlying data.

E.4 The National Import Price Index at Producer Prices

20.190 The data listed in Tables 20.20-20.28 for Industries G, S and T are used to calculate fixed base Laspeyres, Paasche, Fisher and Törnqvist price indices for all imported intermediate inputs (at producer prices or purchase prices in this case), P_L^t, P_P^t, $P_F^{t,}$ and P_T^t, respectively. There are 4 imported intermediate inputs used in Industry G, 4 imported intermediate inputs used in Industry S and 2 imported intermediate inputs used in Industry T so that each import input price index is an aggregate of 10 separate series. The fixed base results are listed in Table 20.48.

Table 20.48 Fixed Base National Import Price Indices at Producer Prices

Period t	P_L^t	P_P^t	P_F^t	P_T^t
1	1.0000	1.0000	1.0000	1.0000
2	1.5210	1.5003	1.5106	1.5089
3	1.2426	1.2037	1.2230	1.2241
4	1.0844	1.0370	1.0604	1.0669
5	1.5776	1.3596	1.4645	1.4736

20.191 The spread between the Laspeyres and Paasche fixed base import price indices is fairly large by period 5, equaling 1.5776/1.3596 = 1.160 or 16.0%. The spread between the Fisher and Törnqvist fixed base indices is much smaller, equaling 1.4736/1.4645 = 1.006 or 0.6% in period 5. Note that each import price index has relatively large period to period fluctuations due to the large fluctuations in the price of imported energy. As usual, we expect the fixed base spreads to diminish if the chained indices are used. The corresponding chained indices are listed in Table 20.49.

Table 20.49 Chained National Import Price Indices at Producer Prices

Period t	P_L^t	P_P^t	P_F^t	P_T^t
1	1.0000	1.0000	1.0000	1.0000
2	1.5210	1.5003	1.5106	1.5089
3	1.2438	1.2384	1.2411	1.2415
4	1.0810	1.0723	1.0766	1.0773
5	1.5128	1.4236	1.4675	1.4680

20.192 Chaining reduces the period 5 spread between Laspeyres and Paasche to 1.5128/1.4236 = 1.063 or 6.3% and between the Fisher and Törnqvist to 1.4680/1.4675 = 1.0003 or 0.03%, a negligible amount.

20.193 The domestic output price index and the domestic export index can be regarded as subindexes of an overall gross output price index of the type that was described in the *PPI Manual*. Similarly, the domestic intermediate input price index and the import price index can be regarded as subindexes of the overall intermediate input price index that was described in the *PPI Manual*. All of these subindices can be thought of as aggregations of the same commodity (or group of commodities) across industries. At a second stage of aggregation, it is possible to aggregate over the domestic output price index and the export price index and to also aggregate over the domestic intermediate input price index and the import price index (with quantities indexed with negative signs) in order to form an economy wide *value added price index*. In the following section, the first stage of aggregation will be across commodities within an industry; i.e., in the following section, industry value added price indices will be constructed. A national value added price index will also be constructed in section E. In section F, the industry value added deflators constructed in section F will be aggregated in order to form a two stage economy wide value added price index. This two stage aggregate value added deflator will be compared with the two stage aggregation method that aggregates over the domestic output price index, the export price index, the domestic intermediate input price index and the import price index. These two methods of two stage aggregation will be compared in section F along with the corresponding single stage national value added deflator.

F. Value Added Price Deflators

F.1 Value Added Price Deflators for the Goods Producing Industry

20.194 The data listed in Tables 20.20-20.22 for Industry G are used to calculate fixed base Laspeyres, Paasche, Fisher and Törnqvist value added price indices or deflators at producer prices. This means that basic prices are used for domestic outputs and exports and purchasers' prices are used for imports and domestic intermediate inputs. The quantities of domestic intermediate inputs and imports are indexed with negative signs. Fixed base and chained value added Laspeyres, Paasche, Fisher and Törnqvist price indices will be constructed, P_L^t, P_P^t, $P_F^{t,}$ and P_T^t, respectively. There are 3 domestic outputs and one export produced by Industry G, and 3 domestic intermediate inputs and 4 imported commodities used as inputs by Industry G so that each value added price index is an aggregate of 11 separate series. The fixed base results are listed in Table 20.50.

Table 20.50 Fixed Base Value Added Price Deflators for Industry G

Period t	P_L^t	P_P^t	P_F^t	P_T^t
1	1.0000	1.0000	1.0000	1.0000
2	1.1655	1.1889	1.1772	1.1535
3	2.2260	3.5528	2.8122	2.5489
4	2.4403	8.0774	4.4398	3.0649
5	1.7605	5.7905	3.1928	2.1276

20.195 The spread between the Laspeyres and Paasche fixed base value added price indices is enormous by period 5, equaling 5.7905/1.7605 = 3.289 or 328.9%. The spread between the Fisher and Törnqvist fixed base indices is large as well, equaling 3.1928/2.1276 = 1.501 or 50.1% in period 5. These very large spreads are due to the fact that the price of high tech services plummets over the sample period with corresponding large increases in quantities while the other prices increase substantially. As well, because quantities have positive and negative weights in value added price indices, the divergences between various index number formulae can become very large. As usual, we expect these spreads to diminish if the chained indices are used. The corresponding chained indices are listed in Table 20.51.

Table 20.51 Chained Value Added Price Deflators for Industry G

Period t	P_L^t	P_P^t	P_F^t	P_T^t
1	1.0000	1.0000	1.0000	1.0000
2	1.1655	1.1889	1.1772	1.1535
3	2.4490	3.2741	2.8317	2.7527
4	2.8776	4.0277	3.4044	3.3096
5	1.8066	2.9594	2.3122	2.2720

20.196 Chaining reduces the period 5 spread in period 5 between Laspeyres and Paasche to 2.9594/1.8066 = 1.638 or 63.8% and between the Fisher and Törnqvist to 2.3122/2.2720 = 1.018 or 1.8%, which is an acceptable degree of divergence considering the volatility of the underlying data. However, note that using the chained Laspeyres or Paasche value added price indices for this industry will give rise to estimates of price change that are very far from the corresponding superlative index estimates. *Thus the corresponding Laspeyres or Paasche estimates of real value added may be rather inaccurate, giving rise to inaccurate estimates of industry productivity growth*.

F.2 Value Added Price Deflators for the Services Industry

20.197 The data listed in Tables 20.23-20.25 for Industry S are used to calculate fixed base Laspeyres, Paasche, Fisher and Törnqvist value added price indices at producer prices. There are 8 domestic outputs and one export produced by Industry S, and 2 domestic intermediate inputs and 4 imported commodities used as inputs by Industry S so that each value added price index is an aggregate of 15 separate series. The fixed base results are listed in Table 20.52. Producer prices are used in these computations.

Table 20.52 Fixed Base Value Added Price Deflators for Industry S

Period t	P_L^t	P_P^t	P_F^t	P_T^t
1	1.0000	1.0000	1.0000	1.0000
2	1.2365	1.2337	1.2351	1.2360
3	1.4876	1.4160	1.4514	1.4537
4	1.5035	1.3531	1.4264	1.4380
5	1.4913	1.2797	1.3814	1.3942

20.198 The spread between the Laspeyres and Paasche fixed base value added price indices for Industry S is 1.4913/1.2797 = 1.165 or 16.5%, which is a substantial gap. The spread between the Fisher and Törnqvist fixed base indices is fairly small, equaling 1.3942/1.3814 = 1.009 or 0.9% in period 5. Note that the gap between the fixed base Paasche and Laspeyres value added price indices for the services industry is very much less than the corresponding gap for the fixed base Paasche and Laspeyres value added price indices for the goods producing industry. An explanation for this narrowing of the Paasche and Laspeyres gap is that while the services industry was subject to some very large fluctuations in the prices it faced, since most of the big fluctuations occurred for the food and energy imports which are margin goods for the industry, these fluctuations were passed on to final demanders, leaving industry distribution margins largely intact. Thus the fluctuations in the value added price indices for Industry S turned out to be less severe than for Industry G. As usual, the spreads between the Paasche and Laspeyres price indices should narrow when the chain principle is used; see Table 20.53 below.

Table 20.53 Chained Value Added Price Deflators for Industry S

Period t	P_L^t	P_P^t	P_F^t	P_T^t
1	1.0000	1.0000	1.0000	1.0000
2	1.2365	1.2337	1.2351	1.2360
3	1.4700	1.4411	1.4555	1.4579
4	1.4620	1.4201	1.4409	1.4432
5	1.4363	1.3863	1.4111	1.4145

20.199 Chaining reduces the period 5 spread between Laspeyres and Paasche to 1.4363/1.3863 = 1.036 or 3.6% in period 5 and between the Fisher and Törnqvist to 1.4145/1.4111 = 1.002, which is negligible.

F.3 Value Added Price Deflators for the Transportation Industry

20.200 The data listed in Tables 20.26-20.28 for Industry T are used to calculate fixed base Laspeyres, Paasche, Fisher and Törnqvist value added price indices at producer prices. There are 3 domestic outputs and one export produced by Industry T, and 3 domestic intermediate inputs and 2 imported commodities used as inputs by Industry T so that each value added price index is an aggregate of 9 separate series. The fixed base results are listed in Table 20.54.

Table 20.54 Fixed Base Value Added Price Deflators for Industry T

Period t	P_L^t	P_P^t	P_F^t	P_T^t
1	1.0000	1.0000	1.0000	1.0000
2	1.4764	1.6417	1.5569	1.5572
3	1.1204	1.1913	1.1553	1.1173
4	1.0977	1.3541	1.2192	1.0679
5	1.8028	4.8128	2.9456	2.2114

20.201 The spread between the Laspeyres and Paasche fixed base value added price indices is enormous by period 5, equaling 4.8128/1.8028 = 2.670 or 267.0%. The spread between the Fisher and Törnqvist fixed base indices is fairly large as well, equaling 2.9456/2.2114 = 1.332 or 33.2% in period 5. These very large spreads are due to the fact that the price of high tech services plummets over the sample period with corresponding large increases in quantities while the other prices increase substantially. As usual, we expect these spreads to diminish if the chained indices are used. The corresponding chained indices are listed in Table 20.55.

Table 20.55 Chained Value Added Price Deflators for Industry T

Period t	P_L^t	P_P^t	P_F^t	P_T^t
1	1.0000	1.0000	1.0000	1.0000
2	1.4764	1.6417	1.5569	1.5572
3	1.0374	1.1271	1.0813	1.0509
4	0.9428	1.0563	0.9979	0.9667
5	1.9916	2.4248	2.1975	2.2389

20.202 Chaining reduces the period 5 spread in period 5 between Laspeyres and Paasche to 2.4248/1.9916 = 1.218 or 21.8% and between the Fisher and Törnqvist to 2.2389/2.1975 = 1.019 or 1.9%, which is an acceptable degree of divergence considering the volatility of the underlying data. However, note that using the chained Laspeyres or Paasche value added price indices for this industry will give rise to estimates of price change that are fairly far from the corresponding chained superlative index estimates, a situation that is similar to what occurred for the Industry G data. Thus whenever possible, it seems preferable to use chained superlative indices when constructing annual industry value added deflators as opposed to using fixed base or chained Paasche or Laspeyres indices.

In the following section, all of the industry data are aggregated to form a national value added deflator.

F.4 The National Value Added Price Deflator

20.203 The data listed in Tables 20.20-20.28 for Industries G, S and T are used to calculate national Laspeyres, Paasche, Fisher and Törnqvist value added price indices at producer prices; i.e., in this subsection, the national value added deflator is constructed. Fixed base and chained value added Laspeyres, Paasche, Fisher and Törnqvist price indices will be constructed, P_L^t, P_P^t, $P_F^{t,}$ and P_T^t, respectively. There are 14 domestic outputs, 3 exported commodities, 8 domestic intermediate inputs and 10 imported commodities so that each national value added deflator is an aggregate of 35 separate series. The fixed base results are listed in Table 20.56.

Table 20.56 Fixed Base National Value Added Deflators

Period t	P_L^t	P_P^t	P_F^t	P_T^t
1	1.0000	1.0000	1.0000	1.0000
2	1.2180	1.2353	1.2267	1.2261
3	1.7776	1.8533	1.8151	1.8173
4	1.8743	1.9822	1.9275	1.9455
5	1.6176	1.7555	1.6851	1.6970

20.204 The spread between the national Laspeyres and Paasche fixed base value added price indices is fairly large by period 5, equaling 1.7555/1.6176 = 1.085 or 8.5%. The spread between the Fisher and Törnqvist fixed base indices is small, equaling 1.6970/1.6851 = 1.007 or 0.7% in period 5. The corresponding chained indices are listed in Table 20.57.

Table 20.57 Chained National Value Added Deflators

Period t	P_L^t	P_P^t	P_F^t	P_T^t
1	1.0000	1.0000	1.0000	1.0000
2	1.2180	1.2353	1.2267	1.2261
3	1.7711	1.8336	1.8021	1.8098
4	1.8855	1.9530	1.9190	1.9315
5	1.6380	1.7612	1.6985	1.7156

20.205 The spread in period 5 between the national Laspeyres and Paasche chained value added price indices equals 1.7612/1.6380 = 1.075 or 7.5% which is slightly smaller than the corresponding 8.5% spread for the fixed base Laspeyres and Paasche indices. The spread between the Fisher and Törnqvist chained indices in period 5 is 1.7156/1.6985 = 1.010 or 1.0%, which is slightly larger than the corresponding fixed base spread of 0.7%. At the national level, the fixed base and chained Fisher and Törnqvist indices all give much the same answer.

G. Two Stage Value Added Price Deflators

G.1 Two Stage National Value Added Price Deflators: Aggregation over Industries

20.206 In section D.6 of chapter 18, methods for constructing a price index by aggregating in two stages were discussed. It was pointed out that if a Laspeyres index is constructed in two stages of aggregation and the Laspeyres formula is used in each stage of aggregation, then the two stage index will necessarily coincide with the corresponding single stage index. A similar consistency in aggregation property holds if the Paasche formula is used at each stage of aggregation. Unfortunately, this consistency in aggregation property does not hold for superlative indices but it was pointed out in chapter 18, that superlative indices should be approximately consistent in aggregation. In this section, the artificial data set will be used in order to evaluate this approximate consistency in aggregation property of the Fisher and Törnqvist indices.

20.207 In the present context, there are two natural ways of aggregating in two stages. In Method 1, the first stage of aggregation is the construction of a value added deflator for each industry (along with the corresponding quantity indices) and in the second stage, the three industry value added deflators are aggregated into a national value added deflator. In Method 2, the first stage of aggregation is the construction of national domestic output, domestic intermediate input, export and import price indices (along with the corresponding quantity indices) and in the second stage, these four price indices are aggregated into a national value added deflator.[57] The results for Method 1 will be listed in this subsection while the results for Method 2 will be listed in section F.2 below.

20.208 In Table 20.58, the fixed base single stage Laspeyres, Paasche, Fisher and Törnqvist indices are listed in the first 4 columns of the table[58] and the corresponding Method 1 fixed base two stage indices are listed in the last 4 columns of the table.

Table 20.58 Fixed Base Single Stage and Two Stage National Value Added Deflators: Aggregation over Industries Method

Period t	P_L^t	P_P^t	P_F^t	P_T^t	P_{L2S}^t	P_{P2S}^t	P_{F2S}^t	P_{T2S}^t
1	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
2	1.2180	1.2353	1.2267	1.2261	1.2180	1.2353	1.2276	1.2190
3	1.7776	1.8533	1.8151	1.8173	1.7776	1.8533	1.8915	1.8110
4	1.8743	1.9822	1.9275	1.9455	1.8743	1.9822	2.2616	1.9254
5	1.6176	1.7555	1.6851	1.6970	1.6176	1.7555	1.9488	1.6579

20.209 As is expected from the theory in Chapter 18, the single stage Laspeyres and Paasche indices coincide exactly with their two stage counterparts. What was not expected is how far the two stage Fisher index, P_{F2S}^t, is from its single stage counterpart, P_F^t, for periods 3-5. Obviously, the period to period changes in the Fisher industry value added indices are so large that the two stage approximation results discussed in Chapter 18 break down for this artificial data set. The spread between the fixed base single stage Fisher and Törnqvist indices in period 5 is 1.6970/1.6851 = 1.007 or 0.7% but the spread between the two stage Fisher and Törnqvist indices in period 5 is 1.9488/1.6579 = 1.175 or 17.5%, a rather large deviation.

20.210 In Table 20.59, the chained single stage Laspeyres, Paasche, Fisher and Törnqvist indices are listed in the first 4 columns of the table[59] and the corresponding Method 1 chained two stage indices are listed in the last 4 columns of the table.

Table 20.59 Chained Single Stage and Two Stage National Value Added Deflators: Aggregation over Industries Method

Period t	P_L^t	P_P^t	P_F^t	P_T^t	P_{L2S}^t	P_{P2S}^t	P_{F2S}^t	P_{T2S}^t
1	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
2	1.2180	1.2353	1.2267	1.2261	1.2180	1.2353	1.2276	1.2190
3	1.7711	1.8336	1.8021	1.8098	1.7711	1.8336	1.8365	1.8124
4	1.8855	1.9530	1.9190	1.9315	1.8855	1.9530	1.9587	1.9326

[57] The domestic output and export quantities are positive numbers in this second stage of aggregation but the domestic intermediate input and import quantities are negative numbers in the second stage of aggregation.

[58] These indices are the same as those listed in Table 20.56.

[59] These indices are the same as those listed in Table 20.57.

5	1.6380	1.7612	1.6985	1.7156	1.6380	1.7612	1.7270	1.7137

20.211 It can be seen that chaining has reduced the spread between the two stage superlative indices. The spread between the chained single stage Fisher and Törnqvist indices in period 5 is 1.7156/1.6985 = 1.007 or 1.0% and the spread between the chained two stage Fisher and Törnqvist indices in period 5 is 1.7270/1.7137 = 1.008 or 0.8%, a rather modest deviation. As is expected from the theory in Chapter 18, the single stage chained Laspeyres and Paasche indices coincide exactly with their two stage counterparts.

G.2 Two Stage National Value Added Price Deflators: Aggregation over Commodities

20.212 In this subsection, the national value added price index is formed by an alternative two stage aggregation procedure. In the first stage aggregation, national domestic output, export, domestic intermediate input and import price indices are calculated along with the corresponding quantity indexes as was done in section F above. In the second stage of aggregation, the sign of the quantity indices that correspond to the domestic intermediate input and import indices is changed from positive to negative and the four price and quantity series are aggregated together to form an estimate for the national value added deflator. The resulting two stage fixed base Laspeyres, Paasche, Fisher and Törnqvist price indices, $P_{L2S}{}^t$, $P_{P2S}{}^t$, $P_{F2S}{}^t$ and $P_{T2S}{}^t$, are listed in the last 4 columns of Table 20.60 along with their fixed base single stage counterparts, $P_L{}^t$, $P_P{}^t$, $P_F{}^t$ and $P_T{}^t$.

Table 20.60 Fixed Base Single Stage and Two Stage National Value Added Deflators: Aggregation over Commodities Method

Period t	$P_L{}^t$	$P_P{}^t$	$P_F{}^t$	$P_T{}^t$	$P_{L2S}{}^t$	$P_{P2S}{}^t$	$P_{F2S}{}^t$	$P_{T2S}{}^t$
1	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
2	1.2180	1.2353	1.2267	1.2261	1.2180	1.2353	1.2272	1.2294
3	1.7776	1.8533	1.8151	1.8173	1.7776	1.8533	1.8067	1.8094
4	1.8743	1.9822	1.9275	1.9455	1.8743	1.9822	1.9066	1.9269
5	1.6176	1.7555	1.6851	1.6970	1.6176	1.7555	1.6641	1.6822

20.213 Note that the single stage fixed base indices, $P_L{}^t$, $P_P{}^t$, $P_F{}^t$ and $P_T{}^t$, listed in Table 20.60 coincide with the single stage fixed base indices $P_L{}^t$, $P_P{}^t$, $P_F{}^t$ and $P_T{}^t$ listed in Table 20.58. Note also that the single stage Paasche and Laspeyres indices coincide with their two stage counterparts in Table 20.60 as is expected from index number theory. Finally, note that the two stage superlative indices, $P_{F2S}{}^t$ and $P_{T2S}{}^t$, are reasonably close to their single stage counterparts, $P_F{}^t$ and $P_T{}^t$. The spread between the four superlative indices is 1.6970/1.6641 = 1.054 or 5.4%. It seems that the Method 2 (aggregation over commodities method) two stage aggregation procedure works more smoothly than the Method 1 (aggregation over industry value added method) two stage aggregation procedure, leading to a reasonably close approximation between the single stage and two stage estimators for the national value added deflator in the case of Method 2.

20.214 In the following Table 20.61, the Method 2 two stage chained Laspeyres, Paasche, Fisher and Törnqvist price indices, P_{L2S}^t, P_{P2S}^t, P_{F2S}^t and P_{T2S}^t, are listed in the last 4 columns of along with their fixed base single stage counterparts, P_L^t, P_P^t, P_F^t and P_T^t.

Table 20.61 Chained Single Stage and Two Stage National Value Added Deflators: Aggregation over Commodities Method

Period t	P_L^t	P_P^t	P_F^t	P_T^t	P_{L2S}^t	P_{P2S}^t	P_{F2S}^t	P_{T2S}^t
1	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
2	1.2180	1.2353	1.2267	1.2261	1.2180	1.2353	1.2272	1.2294
3	1.7711	1.8336	1.8021	1.8098	1.7711	1.8336	1.8037	1.8150
4	1.8855	1.9530	1.9190	1.9315	1.8855	1.9530	1.9202	1.9318
5	1.6380	1.7612	1.6985	1.7156	1.6380	1.7612	1.7069	1.7186

20.215 As expected, chaining reduces the spread between the superlative indices. The spread between the four superlative indices is now 1.7186/1.6985 = 1.012 or 1.2%. Note also that the single stage Paasche and Laspeyres chained indices coincide with their two stage counterparts in Table 20.61.

In the following section, the focus shifts from industry price indices to final demand price indices.

H. Final Demand Price Indices

H.1 Domestic Final Demand Price Indices

20.216 In this section, the standard fixed base and chained Laspeyres, Paasche, Fisher and Törnqvist price indices are listed for deliveries of commodities to the domestic final demand sector; see Table 20.62 below. Each index is an aggregate of 6 separate final demand series.

Table 20.62 Fixed Base and Chained Domestic Final Demand Deflators

	Fixed	Base	Indices		Chained		Indices	
Period t	P_L^t	P_P^t	P_F^t	P_T^t	P_L^t	P_P^t	P_F^t	P_T^t
1	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
2	1.3967	1.3893	1.3930	1.3935	1.3967	1.3893	1.3930	1.3935
3	1.4832	1.4775	1.4803	1.4807	1.4931	1.4952	1.4941	1.4934
4	1.5043	1.4916	1.4980	1.5048	1.5219	1.5219	1.5219	1.5205
5	1.7348	1.6570	1.6954	1.7108	1.7176	1.7065	1.7120	1.7122

20.217 The indices listed in Table 20.62 have already been listed in various tables in section C above but for convenience, they are tabled again. Since the above indices have been discussed in section C, the discussion will not be repeated here.

H.2 Export Price Indices at Final Demand Prices

20.218 In this subsection, the standard fixed base and chained Laspeyres, Paasche, Fisher and Törnqvist price indices are calculated for the 3 export series that are listed in section B above. Final demand prices are used when calculating the indices listed in Table 20.63 below.

Table 20.63 Fixed Base and Chained Export Price Indices at Final Demand Prices

	Fixed	**Base**	**Indices**		**Chained**		**Indices**	
Period t	P_L^t	P_P^t	P_F^t	P_T^t	P_L^t	P_P^t	P_F^t	P_T^t
1	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
2	1.3191	1.321	1.3201	1.3202	1.3191	1.3210	1.3201	1.3202
3	1.5816	1.5789	1.5802	1.5803	1.5775	1.5777	1.5776	1.5776
4	1.4752	1.4750	1.4751	1.4750	1.4703	1.4716	1.4709	1.4709
5	1.4184	1.4152	1.4168	1.4167	1.4140	1.4076	1.4108	1.4105

20.219 Since the 3 export price and quantity series have fairly smooth trends that are roughly proportional to each other, all of the indices listed above in Table 20.63 are quite close to each other.

H.3 Import Price Indices at Final Demand Prices

20.220 In this subsection, the standard fixed base and chained Laspeyres, Paasche, Fisher and Törnqvist price indices are calculated for the 10 import series that are listed in section B above. Final demand prices are used when calculating the indices listed in Table 20.64 below.

Table 20.64 Fixed Base and Chained Import Price Indices at Final Demand Prices

	Fixed	**Base**	**Indices**		**Chained**		**Indices**	
Period t	P_L^t	P_P^t	P_F^t	P_T^t	P_L^t	P_P^t	P_F^t	P_T^t
1	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
2	1.5495	1.5279	1.5387	1.5369	1.5495	1.5279	1.5387	1.5369
3	1.2270	1.1907	1.2087	1.2099	1.2293	1.2261	1.2277	1.2285
4	1.0739	1.0289	1.0512	1.0580	1.0709	1.0642	1.0676	1.0682
5	1.5946	1.3726	1.4794	1.4873	1.5257	1.4321	1.4782	1.4785

20.221 Since price and quantity trends for imports are far from being proportional, there are substantial differences between the Paasche and Laspeyres price indices. The spread between the fixed base Paasche and Laspeyres is 1.5946/1.3726 = 1.162 or 16.2% while the spread between the chained Paasche and Laspeyres is 1.5257/1.4321 = 1.065 or 6.5% so that as usual, chaining reduces the spread. All of the superlative indices are close to each other.

H.4 GDP Deflators

20.222 In this subsection, various GDP deflators are calculated; i.e., the standard fixed base and chained Laspeyres, Paasche, Fisher and Törnqvist price indices are calculated for the 19 final demand series that are listed in section B above. Final demand prices are used when calculating the indices listed in Table 20.65 below.

Table 20.65 Fixed Base and Chained GDP Deflators

	Fixed	**Base**	**Indices**		**Chained**		**Indices**	
Period t	P_L^t	P_P^t	P_F^t	P_T^t	P_L^t	P_P^t	P_F^t	P_T^t
1	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
2	1.2376	1.2482	1.2429	1.2417	1.2376	1.2482	1.2429	1.2417
3	1.7317	1.7696	1.7506	1.7546	1.7252	1.7632	1.7441	1.7499
4	1.8107	1.8476	1.8291	1.8488	1.8139	1.8507	1.8322	1.8420
5	1.6591	1.7044	1.6816	1.6995	1.6581	1.7391	1.6981	1.7099

20.223 The spread between the Paasche and Laspeyres fixed base GDP deflators in period 5 is 1.7044/1.6591 = 1.027 or 2.7% while the spread between the Paasche and Laspeyres chained GDP deflators in period 5 is 1.7044/1.6591 = 1.048 or 4.8%. Thus in this case, chaining did not reduce the spread between the Paasche and Laspeyres indices. The superlative indices are all rather close to each other; in period 5, the spread between the 4 superlative indices was 1.7099/1.6816 = 1.017 or 1.7% and the spread between the two chained superlative indices was only 1.7099/1.6981 = 1.007 or 0.7%.

H.5 The Reconciliation of the GDP Deflator with the Value Added Deflator

20.224 The final set of tables for this chapter draws on the theory developed in section B.3 of Chapter 18. In that section, it was shown how volume estimates for GDP at final demand prices, GDP_F, could be reconciled with volume estimates for GDP at producer prices, GDP_P, using equation (17.27). Equation (17.27) said that GDP_F equals GDP_P plus a sum of tax terms, T. In Chapter 18, it was shown that two stage price and quantity indices for GDP_F could be constructed by aggregating over the 35 separate price and quantity series that are used to construct price and quantity indices for GDP_P plus aggregating over all of the tax series that make up the T aggregate. It was shown in Chapter 18 that the resulting price and volume estimates for GDP_F and GDP_P + T will coincide if the Laspeyres, Paasche or Fisher formulae are used. This methodology is tested out on the artificial data set for both fixed base and chained Laspeyres, Paasche, Fisher and Törnqvist price indices in Tables 20.66 (fixed base indices) and 20.67 (chained indices) below. The P_L^t, P_P^t, P_F^t and P_T^t indices reported in Table 20.66 are the fixed base single stage GDP deflators (for GDP_F) that were listed in the first 4 columns of Table 20.65 while the P_{L2S}^t, P_{P2S}^t, P_{F2S}^t and P_{T2S}^t indices reported in Table 20.66 are the two stage fixed base price indices that result when we aggregate over the 35 component price and quantity series that make up GDP at producer prices, GDP_P, plus the nonzero tax series that are listed in section B above and make up the tax aggregate T.

Table 20.66 Fixed Base GDP Deflators Calculated in Two Stages

Period t	P_L^t	P_P^t	P_F^t	P_T^t	P_{L2S}^t	P_{P2S}^t	P_{F2S}^t	P_{T2S}^t
1	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
2	1.2376	1.2482	1.2429	1.2417	1.2376	1.2482	1.2429	1.2428
3	1.7317	1.7696	1.7506	1.7546	1.7317	1.7696	1.7506	1.7538
4	1.8107	1.8476	1.8291	1.8488	1.8107	1.8476	1.8291	1.8470
5	1.6591	1.7044	1.6816	1.6995	1.6591	1.7044	1.6816	1.7020

20.225 As predicted by the theory presented in Chapter 18, the Laspeyres, Paasche and Fisher single stage estimates for the GDP deflator (the first 3 columns in Table 20.66) coincide exactly with the corresponding two stage estimates that are built up by aggregating over GDP at producer prices plus aggregating over the tax series. The single stage Törnqvist GDP deflator, P_T^t, does not coincide with its two stage counterpart, P_{T2S}^t, but the correspondence is fairly close.

20.226 The P_L^t, P_P^t, P_F^t and P_T^t indices reported in Table 20.67 are the chained single stage GDP deflators (for GDP_F) that were listed in the last 4 columns of Table 20.65 while the P_{L2S}^t, P_{P2S}^t, P_{F2S}^t and P_{T2S}^t indices reported in Table 20.67 are the two stage chained price indices that result when we aggregate over the 35 component price and quantity series that make up GDP at producer prices, GDP_P, plus the nonzero tax series that are listed in section B above and make up the tax aggregate T.

Table 20.67 Chained GDP Deflators Calculated in Two Stages

Period t	P_L^t	P_P^t	P_F^t	P_T^t	P_{L2S}^t	P_{P2S}^t	P_{F2S}^t	P_{T2S}^t
1	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
2	1.2376	1.2482	1.2429	1.2417	1.2376	1.2482	1.2429	1.2428
3	1.7252	1.7632	1.7441	1.7499	1.7252	1.7632	1.7441	1.7488
4	1.8139	1.8507	1.8322	1.8420	1.8139	1.8507	1.8322	1.8405
5	1.6581	1.7391	1.6981	1.7099	1.6581	1.7391	1.6981	1.7120

20.227 Again as predicted by the theory presented in Chapter 18, the Laspeyres, Paasche and Fisher single stage estimates for the GDP deflator (the first 3 columns in Table 20.67) coincide exactly with the corresponding two stage estimates that are built up by aggregating over GDP at producer prices plus aggregating over the tax series. The single stage Törnqvist GDP deflator, P_T^t, does not coincide with its two stage counterpart, P_{T2S}^t, but again, the correspondence is fairly close.

20.228 The equality of the single stage and two stage Laspeyres, Paasche and Fisher GDP_F deflators in Tables 20.66 and 20.67 provides a very good check on the correctness of all of the various index number calculations that are associated with PPI programs and the production of GDP volume estimates.

I. Conclusion

20.229 Some tentative conclusions that can be drawn from the various indices that have been computed using the artificial data set are as follows:

- It is risky to use fixed base Paasche or Laspeyres indices in the sense that they can be rather far from the theoretically preferred superlative indices.
- Chained indices seem preferable to the use of fixed base indices in the sense that chaining generally reduces the spread between the Paasche and Laspeyres indices.
- Chained Paasche and Laspeyres indices can be close to the theoretically preferred superlative indices, except in the value added context; i.e., chained Paasche and Laspeyres indices are often fairly close to each other (and the corresponding chained superlative indices) when constructing output, export, intermediate input and import price indices. However, when constructing value added indices, it seems preferable to use chained superlative indices.

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